Example 7.2 Rectangular foundation subjected to eccentric loading

1 Description of the problem

For comparison with complex foundation shape, no analytical solution is yet available. Therefore, for judgment on the nonlinear analysis of foundations for simple assumption model, consider the rectangular foundation shown in Figure 7.5. The foundation has the length L = 8.0 [m] and the width B = 6.0 [m]. The foundation carries an eccentric load of N = 2000 [kN]. Both of the x-axis and y-axis are main axes, which intersect in the center of gravity s of the foundation area. The position of resultant N is defined by the ordinates $x = e_x$ and $y = e_y$. Within the rectangle foundation area five zones are represented. It is found that the contact area and maximum corner pressure $max\ q_o$ depends on the position of the resultant N in these five zones (*Irles*/ *Irles* (1994)). In this example, the maximum corner pressure $max\ q_o$ is obtained using the program *ELPLA* for each zone and compared with other analytical salutations, which are available for rectangular foundation.

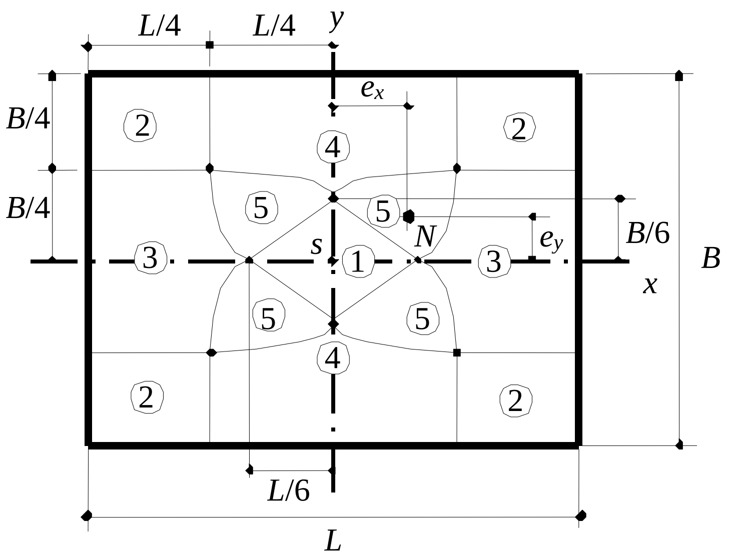

Figure 7.5 Division of the rectangular foundation in five zones according to the position of the resultant N

2 Hand calculation of the maximum corner pressure *max* q_o

The maximum corner pressure *max* q_o for the zone (1) can be obtained directly using Equation 7.1, where in this case the Resultant *N* lies in the foundation kern and no separation will occur. The maximum corner pressure *max* q_o for the other four zones can be obtained using available analytical solutions according to *Irles*/ *Irles* (1994), *Teng* (1962) and *Graßhoff*/ *Kany* (1997) as follows:

Zone (2)
Three corners detached (e_x = 3.0 [m], e_y = 2.25 [m])

The maximum corner pressure *max* q_o for zone (2), Figure 7.6a, can be given according to *Irles*/ *Irles* (1994) from the following equation

$$\max q_o = \frac{3N}{2\left(L-2e_x\right)\left(B-2e_y\right)}$$

$$\max q_o = \frac{3\times 2000}{2\left(8-2\times 3\right)\left(6-2\times 2.25\right)} = 1000 [\text{kN/m}^2]$$

Zone (3)
Two corners detached (e_x = 3.0 [m], e_y = 0.0 [m])

The maximum corner pressure *max* q_o for zone (3), Figure 7.6b, can be given according to *Teng* (1962) from the following equation

$$\max q_o = \frac{N}{L\,B}\left(\frac{4L}{3L-6e_x}\right)$$

$$\max q_o = \frac{2000}{8\times 6}\left(\frac{4\times 8}{3\times 8-6\times 3}\right) = 222.22 [\text{kN/m}^2]$$

Zone (4)
Two corners detached (e_x = 1.0 [m], e_y = 2.25 [m])

The maximum corner pressure *max* q_o for zone (4), Figure 7.6c, can be given according to *Graßhoff*/ *Kany* (1997) from the following equation

$$t = \frac{L}{12}\left(\frac{L}{e_x}+\sqrt{\frac{L^2}{e_x^{\,2}}-12}\right) = \frac{8}{12}\left(\frac{8}{1.0}+\sqrt{\frac{8^2}{1.0^2}-12}\right) = 10.141 [\text{m}]$$

$$\tan\beta = \frac{3}{2}\left(\frac{B-2e_x}{t+e_x}\right) = \frac{3}{2}\left(\frac{6-2\times 2.25}{10.141+1.0}\right) = 0.202$$

$$\max q_o = \frac{12N}{L\tan\beta}\,\frac{L+2t}{L^2+12t^2} = \frac{12\times 2000}{8\times 0.202}\,\frac{8+2\times 10.141}{8^2+12\times 10.141^2} = 323.58 [\text{kN/m}^2]$$

Zone (5)
Only one corner detached (e_x = 1.0 [m], e_y = 0.75 [m])

The maximum corner pressure *max* q_o for zone (5), Figure 7.6d, can be given according to *Graßhoff*/ *Kany* (1997) from the following equation

$$K = \frac{e_x}{L} + \frac{e_y}{B} = \frac{1.0}{8} + \frac{0.75}{6} = 0.25$$

$$\max q_o = \frac{N}{L\,B} K\left[12 - 3.9\left(6\,K - 1\right)\left(1 - 2\,K\right)\left(2.3 - 2\,K\right)\right]$$

$$\max q_o = \frac{2000}{8 \times 6} K\left[12 - 3.9\left(6 \times 0.25 - 1\right)\left(1 - 2 \times 0.25\right)\left(2.3 - 2 \times 0.25\right)\right]$$

$$\max q_o = 106.72\,[\text{kN/m}^2]$$

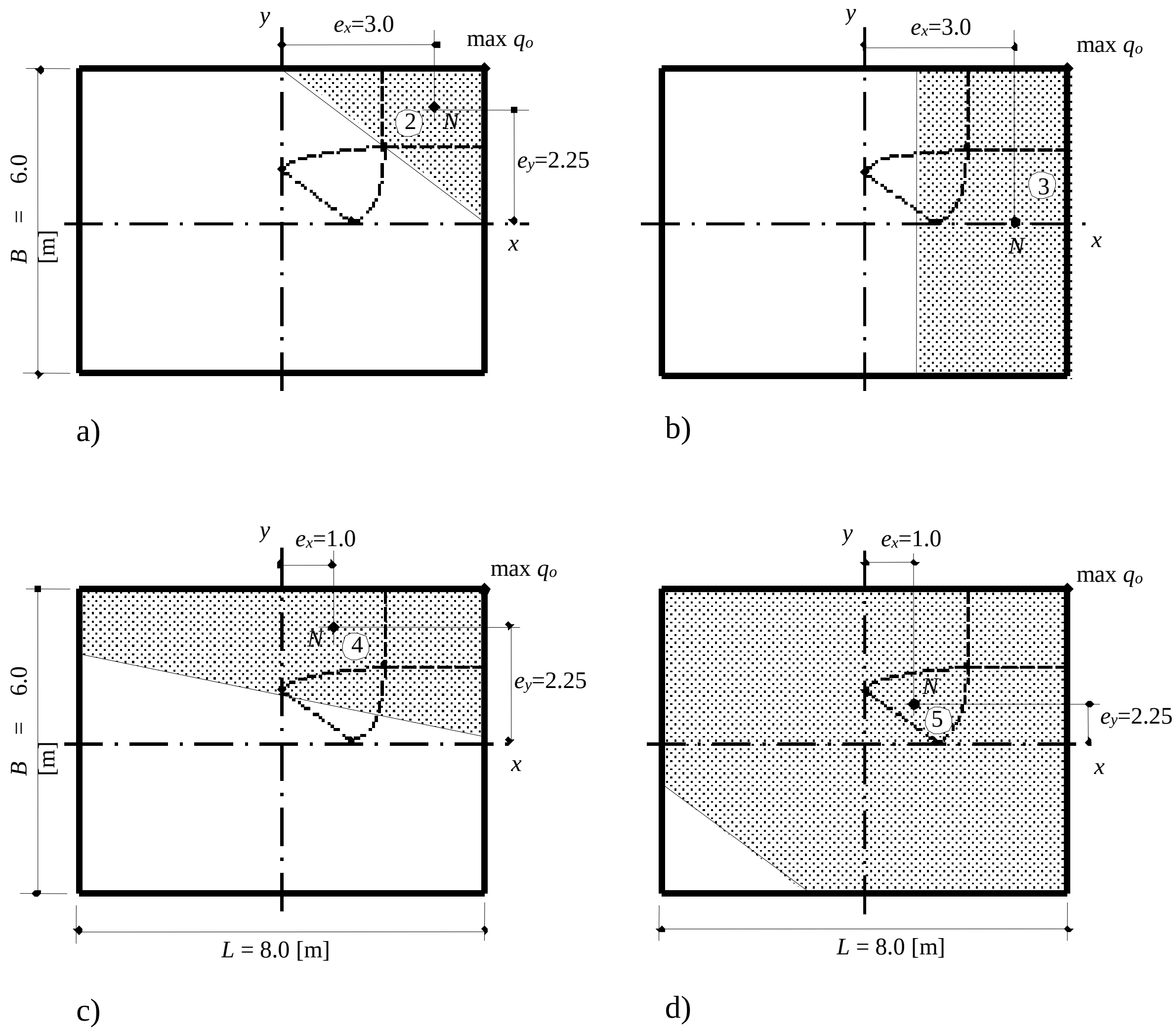

Figure 7.6 Resultant *N* lies in the four zones (2) to (5)

3 Determination of the maximum corner pressure *max* q_o by the program *ELPLA*

To achieve the comparison between the maximum corner pressure *max* q_o obtained from the program *ELPLA* and that obtained from the other available analytical solutions described above, the rectangular foundation is subdivided into refine mesh of square finite elements. Each element has a side of 0.1 [m]. The results obtained from the program *ELPLA* are compared with those obtained above in Table 7.3. It shows that the results of both the analytical and iteration methods are in a good agreement.

Table 7.3 Comparison between the maximum corner pressure *max* q_o [kN/m^2] obtained from program *ELPLA* and that obtained from the available analytical solutions

Zone No.	Zone (2)	Zone (3)	Zone (4)	Zone (5)
Available solutions	*Irles/ Irles* (1994)	*Teng* (1962)	*Graßhoff/ Kany* (1997)	
	1000	222	324	107
ELPLA	1017	223	325	106
Difference [%]	1.67	0.45	0.31	0.94