Example 6.1 Analysis of a raft for a high rise building

1 Description of the problem

This example was carried out to show the influence of flexure rigidity of the superstructure on the settlements and contact pressures for a raft of high rise building.

It is required to analyze a raft for the building shown in Figure 6.6 in three simplified sections. The building is a reinforced concrete skeleton structure and consists of a cellar and 13 storeys. The floor height is 3 [m] while the bay width is 3.6 [m]. The number of bays is 18. The total building length is 66 [m] while the total width of the cellar basement is 17.55 [m]. The raft thickness is 1.2 [m]. In the following study the raft is analyzed considering subsoil behavior. Also, an estimation of the superstructure deformations is carried out. In the analysis, settlements and contact pressures are determined in which a comparison is carried out in four cases as:

i) For not stiffened raft
ii) For compound system raft-cellar
iii) For compound system raft-cellar-superstructure
iv) For completely rigid raft

The stiffness of the structure system parallel to the long axis can be determined from the data given in Figures 6.6 and 6.7.

2 Soil properties

According to Figure 6.7, the subsoil layers consist of a sandy clay layer until 11.6 [m] depth under the ground surface with modulus of compressibility E_s = 14 000 [kN/m^2]. Under the sandy clay layer exists in 11.60 [m] depth practically incompressible sandstone rock in great thickness. The settlement parts from the reloading of the soil are neglected. The foundation level under the original ground surface is 3.80 [m]. The modulus of compressibility method is used to analyze the foundation.

3 Material properties of concrete

The building material is reinforced concrete and has the following properties:

Young's modulus	E_b	$= 2 \times 10^7$	[kN/m^2]
Poisson's ratio	ν_b	= 0.25	[-]
Unit weight	γ_b	= 0.0	[kN/m^3]

Unit weight of the concrete is chosen γ_b = 0.0 to neglect the self-weight of the structure.

4 Loads

According to static calculation of the open frame assuming rigid supports, each column from the twice 17 columns of the external walls has a column load of 2700 [kN] while each column of the twice 17 internal columns has a column load of 2500 [kN]. The column load for the four corner columns is 1350 [kN] while for the four edge columns is 1250 [kN]. The loads with FE-Net of the raft are shown in Figure 6.8.

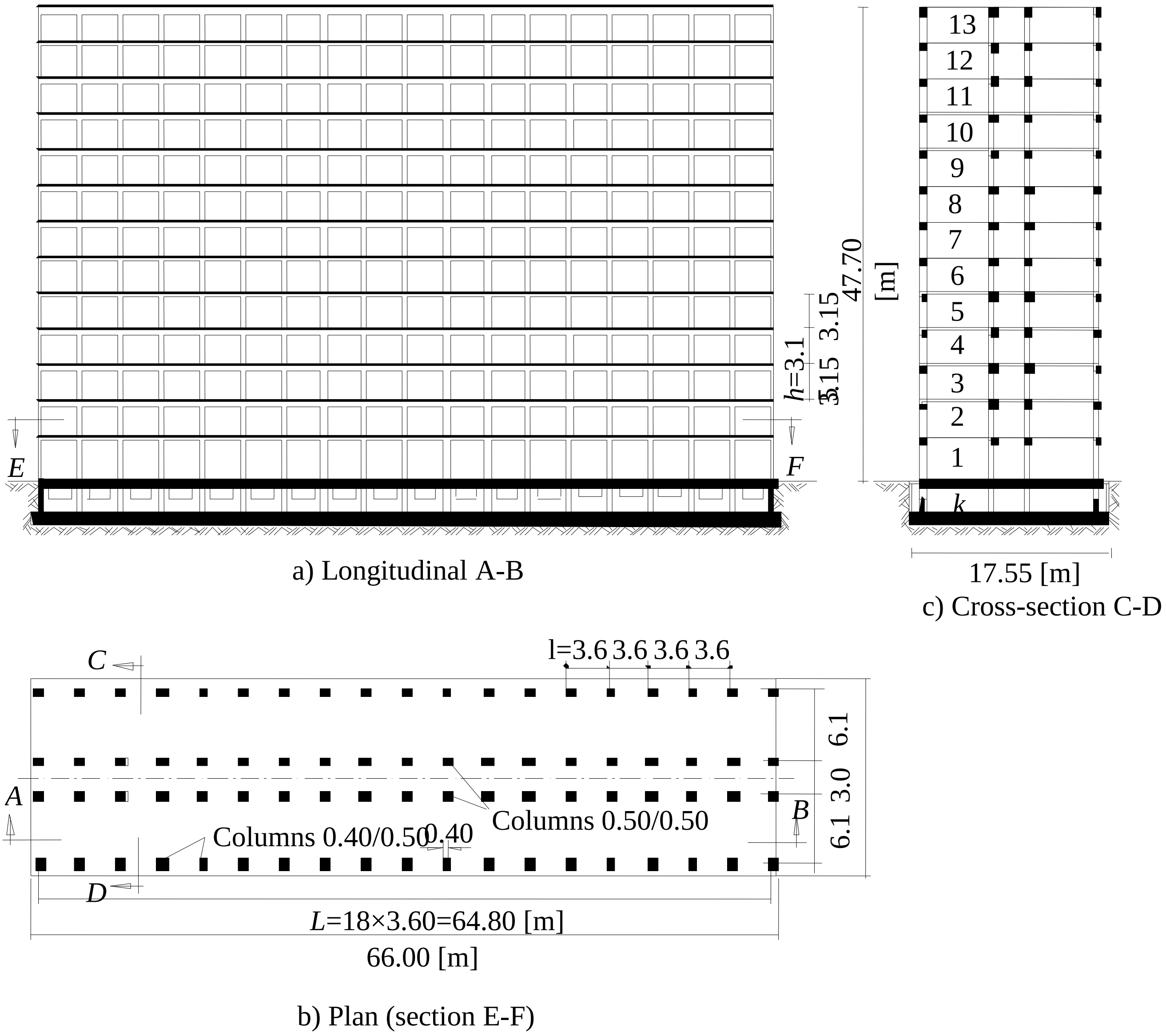

Figure 6.6 Details of the building

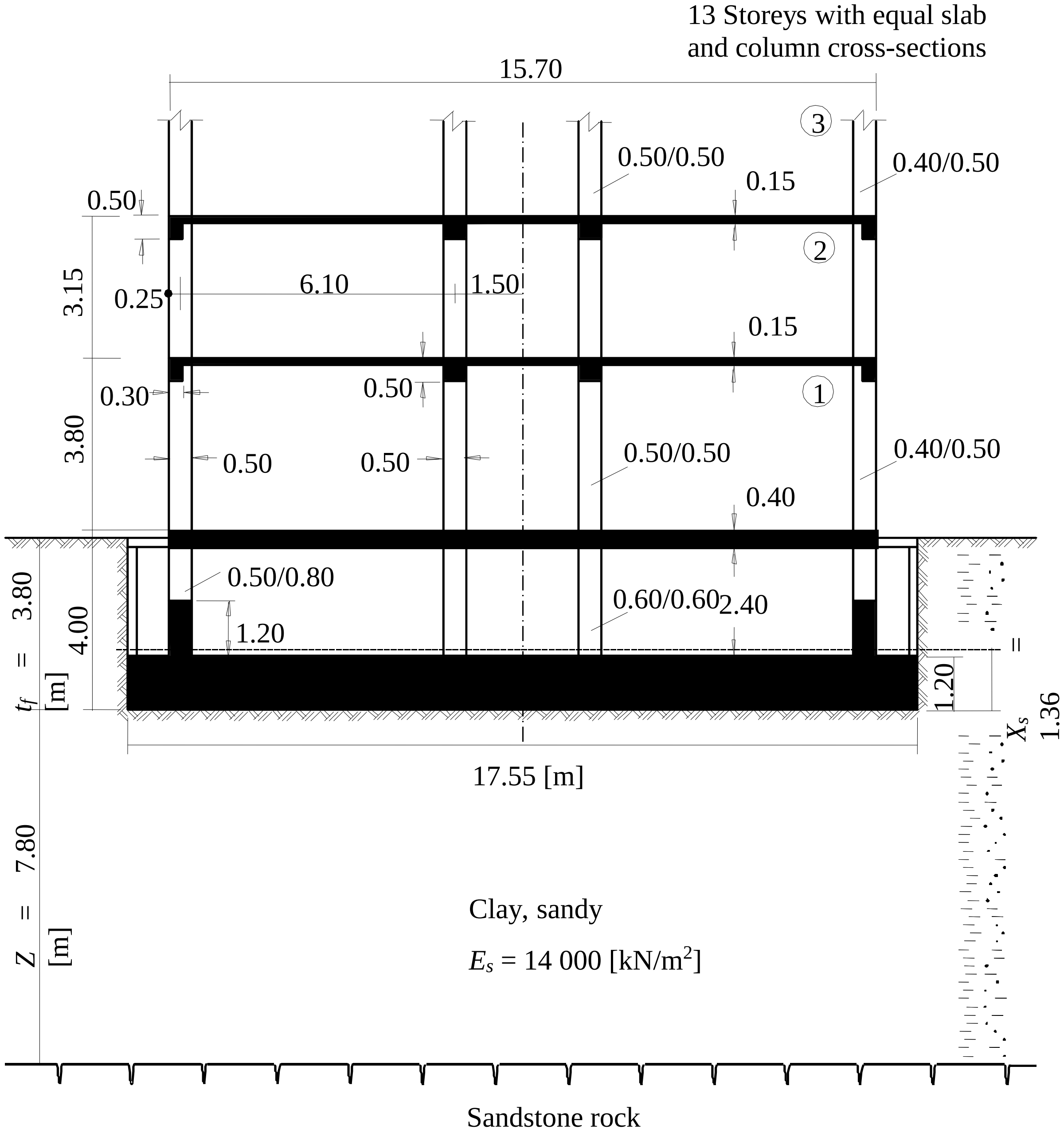

Figure 6.7 Subsoil and dimensions of the raft, floors and columns (cross-section)

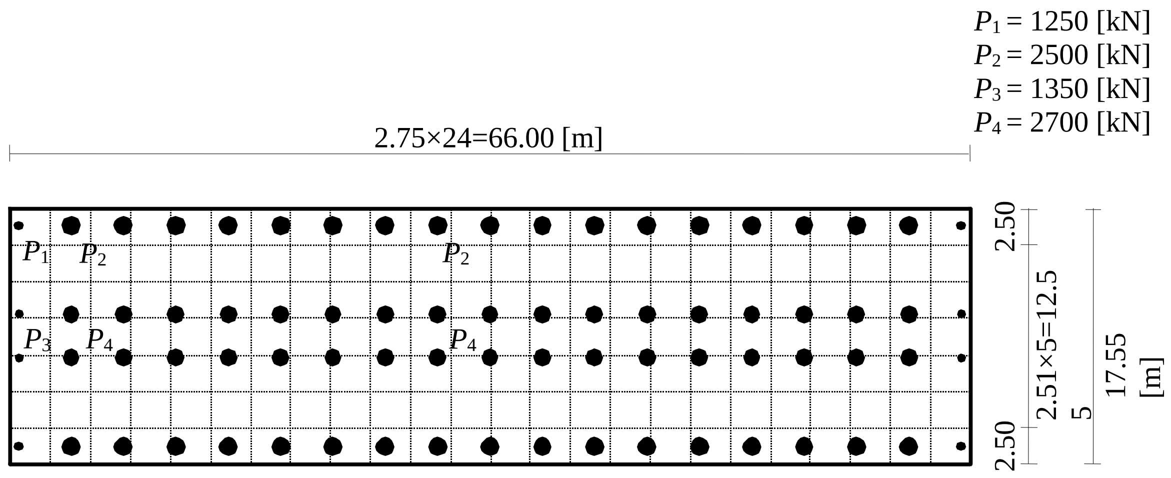

Figure 6.8 FE-Net of the raft with loads

5 Analysis of the structure

5.1 Analysis for not stiffened raft

At first, the settlements and contact pressures are determined under the assumption that except for the stiffness of the raft itself (thickness d = 1.2 [m]) no other rigidity is effective. So, the flexure rigidity of the raft K_G can be obtained from

$$K_G = E_G\, I_G = E_G\, \frac{B\, d^3}{12} = 2.1\times 10^7\, \frac{17.55(1.2)^3}{12} = 5.31\times 10^7 \text{ [kN/m}^3\text{]}$$

and system rigidity K_{st}

$$K_{st} = 12\, K_s = \frac{E_b}{E_s}\left(\frac{d}{L_f}\right)^3 = \frac{2.1\times 10^7}{14000}\left(\frac{1.2}{66}\right)^3 = 0.009[-]$$

The raft is flexible according to Table 6.1, (0.01 > K_{st}).

5.2 Analysis for the compound system raft-cellar

From the assumption that the raft, the cellar walls and the cellar thickness represent combined flexure rigidity for the cross section, the cellar system with the raft must be connected rigidly through satisfied reinforcement. Considering the cross-section shown in Figure 6.7, the height x_s of the center of gravity of the system cellar-raft is given by

$$x_s = \frac{\sum F_i\, x_i}{\sum F_i} = \frac{(17.55\times 1.2\times 0.6)+(2\times 0.5\times 1.2\times 1.8)+(0.4\times 15.7\times 3.8)}{(17.55\times 1.2)+(2\times 0.5\times 1.2)+(0.4\times 15.7)} = 1.36\,[\text{m}]$$

Then, the moment of inertia I_G of the foundation system according to *Steiner*'s law is given by

$$I_G = \left(\frac{17.55\times(1.2)^3}{12} + 17.55\times 1.2\times(0.76)^2\right) + 2\left(\frac{0.5\times(1.2)^3}{12} + 0.5\times 1.2\times(0.44)^2\right)$$
$$+\left(\frac{15.70\times(0.4)^3}{12} + 15.7\times 0.4\times(2.44)^2\right) = 52.54\,[\text{m}^4]$$

The rigidity of the structure K_G is given by

$$K_G = E_G\, I_G = 2.1\times 10^7 \times 52.4 = 110.33\times 10^7\ [\text{kN/m}^3]$$

Then, the ideal raft thickness d_i is given by

$$d_i = \sqrt[3]{\frac{12\,I}{B}} = \sqrt[3]{\frac{12\times 52.54}{17.55}} = 3.3\,[\text{m}]$$

and system rigidity K_{st}

$$K_{st} = 12\,K_s = \frac{E_b}{E_s}\left(\frac{d}{L_f}\right)^3 = \frac{2.1\times 10^7}{14000}\left(\frac{3.3}{66}\right)^3 = 0.1875\,[\text{-}]$$

The raft is stiff according to Table 6.1, ($0.2 > K_{St} \geq 0.1$).

5.3 Analysis for the compound system raft-cellar-superstructure

In this case the structure system is considered as a raft, cellar and superstructure connected together as one unit. Here, the statical system of the structure may be taken as multi-storey open frame (13 storeys, 18 bays), which is statically indeterminate. The next calculation shows a simplificative way to estimate the rigidity of the overall structure on the foundation. In the calculation, it is assumed that only the rigidity of the open panels is taken into consideration where the contribution of filling walls on the structure rigidity is neglected.

Moment of inertia of the floor I_r

According to Beton-Kalender (1957), page 47 or *El Behairy* (1992), page 17 the moment of inertia can be obtained from

$$\frac{\sum b_o}{b} = \frac{2\,(0.3+0.5)}{15.7} = \frac{1.6}{15.7} = 0.102,$$

$$\frac{d}{d_o} = \frac{0.15}{0.5} = 0.3,$$

$$\mu = 0.0193$$

$$I_r = \mu\, b\, d_o^{\,3} = 0.0193 \times 15.7 \times (0.5)^3 = 0.0379[\mathrm{m}^3]$$

Average stiffness of the floor K_r

$$K_r = \frac{I_r}{l} = \frac{0.0379}{3.6} = 0.01053[\mathrm{m}^3]$$

Moment of inertia of the columns I_s

The columns consist of two internal columns with cross-section of 0.5 × 0.5 [m] and two external columns with cross-section of 0.5 × 0.4 [m].

$$I_s = 2\left(\frac{0.5 \times 0.5^3}{12} + \frac{0.5 \times 0.4^3}{12}\right) = 0.01575[\mathrm{m}^4]$$

Average stiffness of the columns K_s

$$K_s = \frac{I_s}{h} = \frac{0.01575}{3.15} = 0.005[\mathrm{m}^3]$$

Since all floors and columns are supposed to have similar cross-sections, the effective moment of inertia I_B of the multi-storey open frame according to *Meyerhof* (1953) can be given by

$$I_B = I_r\, n_s\, n_s^{\,2}\, \frac{2\,K_s}{K_r + 2\,K_s} = 0.0379 \times 13 \times 18^2\, \frac{2 \times 0.005}{0.01053 + 2 \times 0.005} = 77.76\,[\mathrm{m}^4]$$

Flexure rigidity of the superstructure K_B

$$K_B = E_B\, I_B = 2.1 \times 10^7 \times 77.76 = 163.29 \times 10^7\ [\mathrm{kN/m}^3]$$

Flexure rigidity of the entire structure K_b

$$K_b = K_G + K_B = 110.33 \times 10^7 + 163.29 \times 10^7 = 273.62 \times 10^7\ [\mathrm{kN/m}^3]$$

Ideal moment of inertia for the entire structure *I*

$$I = \frac{K_b}{E_b} = \frac{273.62 \times 10^7}{2.1 \times 10^7} = 130.3\,[\mathrm{m}^4]$$

Ideal raft thickness d_i

$$d_i = \sqrt[3]{\frac{12\,I}{B}} = \sqrt[3]{\frac{12 \times 130.3}{17.55}} = 4.46\,[\mathrm{m}]$$

and system rigidity K_{st}

$$K_{st} = 12\,K_s = \frac{E_b}{E_s}\left(\frac{d}{L_f}\right)^3 = \frac{2.1 \times 10^7}{14000}\left(\frac{4.46}{66}\right)^3 = 0.463[\text{-}]$$

The raft is very stiff according Table 6.1, (1.0 > $K_{St} \geq 0.4$).

5.4 Analysis for completely rigid raft

In this case both the superstructure and foundation are considered as an infinitely rigid structure. To determine the settlements and contact pressures in this extreme case, the modulus of compressibility method for the rigid raft is used. This method considers the raft is completely rigid. Rigid raft means a raft has a thickness of $d = \infty$ which also lead to a flexure rigidity of $K_G = \infty$.

Figures 6.9 and 6.10 show the settlements and contact pressures for the four cases of analyses. The settlements and contact pressures are determined with an ideal raft thickness d_i. Furthermore, the results of this example are represented in Table 6.3 in details, so that one can recognize the differences well.

6 Conclusions

This study shows that the results with and without the influence of the structure rigidity are different from one to other. Besides, the numerical example shows a way to how it can determine for more complicated structure systems the settlements and contact pressures taking into account the influence of the structure rigidity.

Table 6.3 Results of structure rigidity for the four different cases of analyses

Analysis	Moment of inertia I [m^4]	Flexure rigidity $K = E_b I$ [kN/m^3]	Ideal raft thickness d_i [m]	System rigidity K_{st} [1]	Grade of System rigidity
Not stiffened raft	2.53	5.31×10^7	1.20	0.009	Flexible $0.01 > K_{st}$
Compound system raft-cellar	52.54	110.33×10^7	3.30	0.1875	Stiff $0.2 > K_{St} \geq 0.1$
Compound system raft-cellar-superstructure	130.30	273.62×10^7	4.46	0.463	Very stiff $1.0 > K_{St} \geq 0.4$
Completely rigid raft	∞	∞	∞	∞	Rigid $K_{St} \geq 1.0$

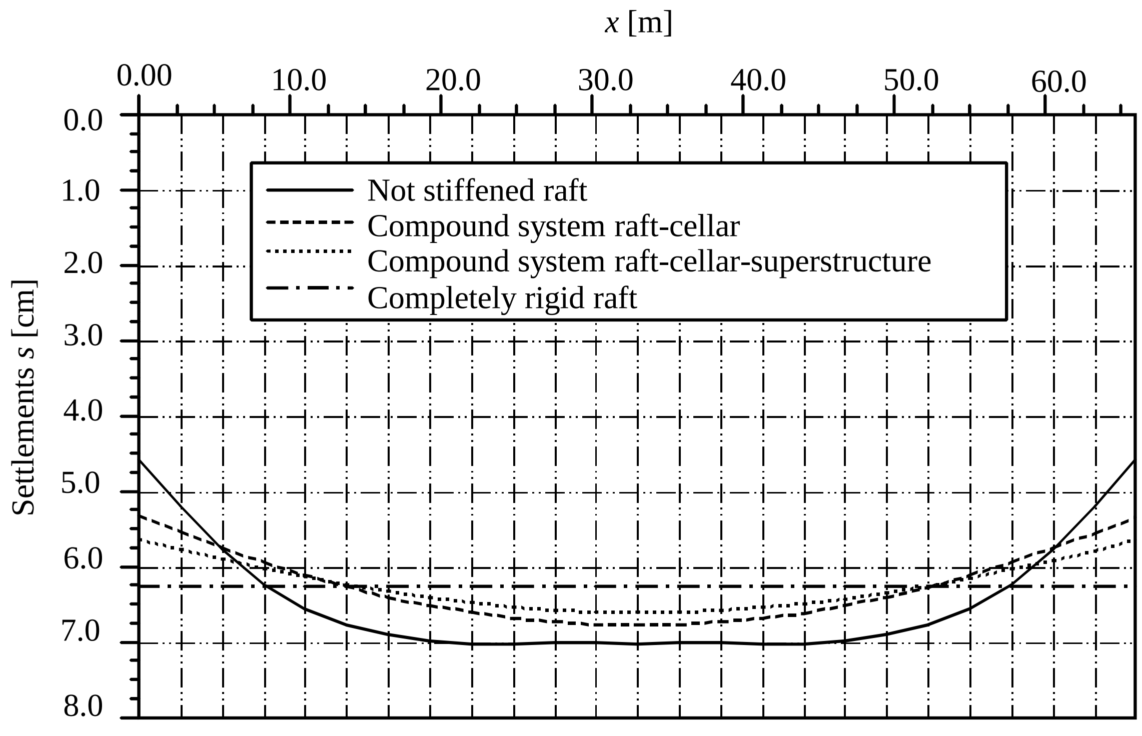

Figure 6.9 Settlements s [m] in longitudinal direction at the middle of the structure

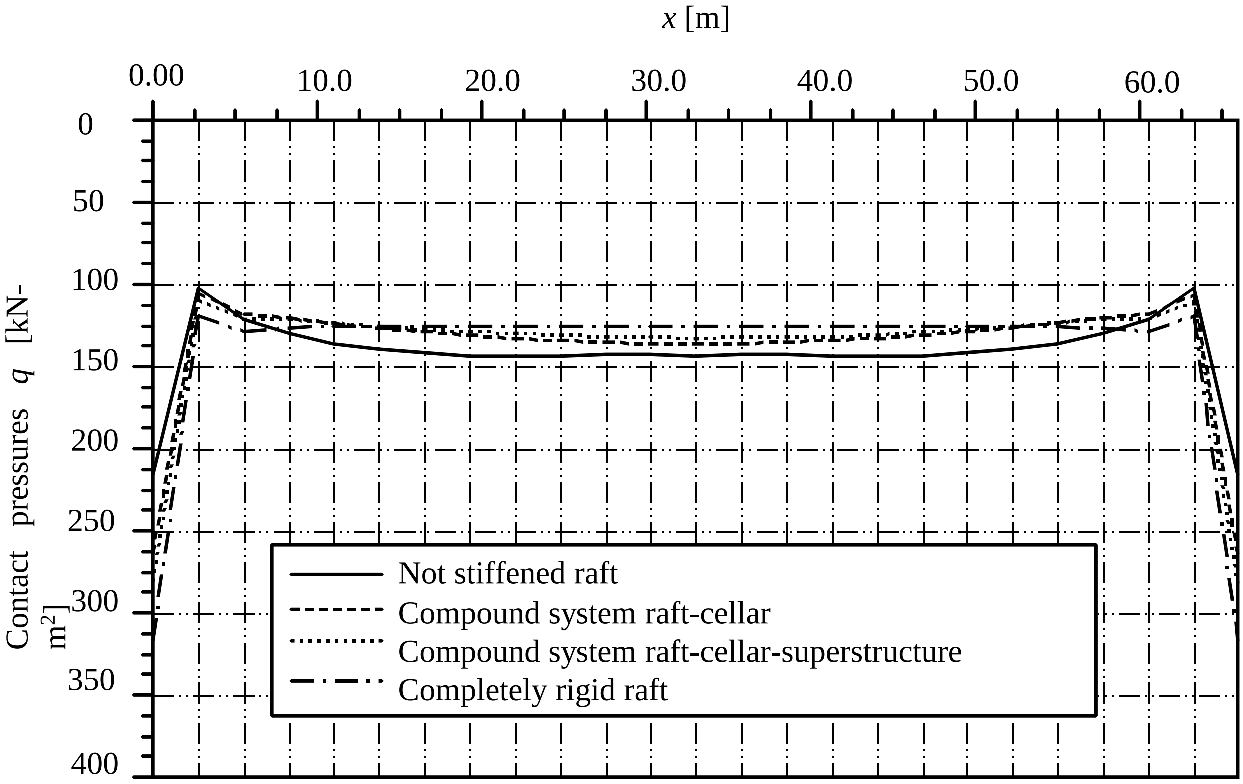

Figure 6.10 Contact pressures q [kN/m^2] in longitudinal direction at the middle of the structure