Consolidation Settlement by the Program *GEO Tools*

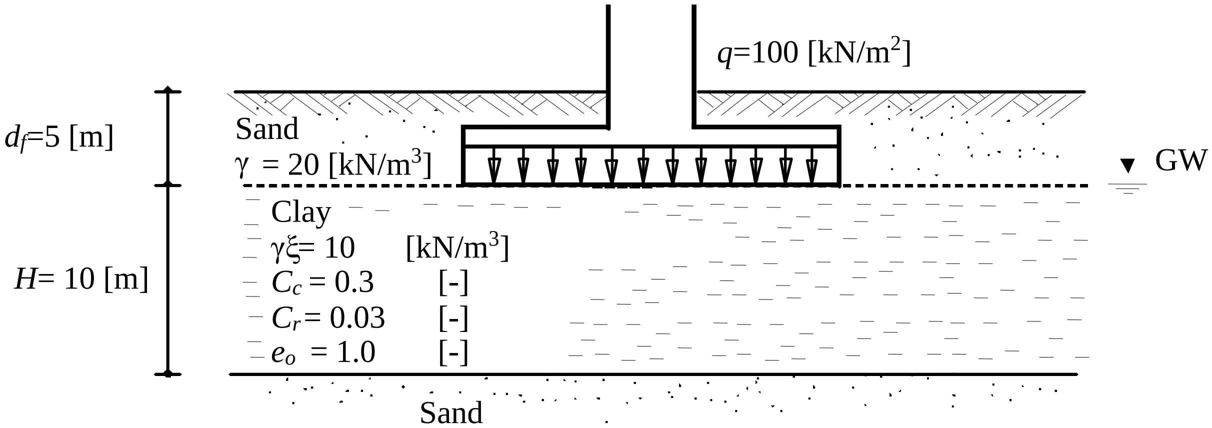

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Table of Contents

Preface

Various problems in geotechnical Engineering can be investigated by the program *GEO Tools*. The original version of the program *GEO Tools GEOTEC Office* was developed by Prof. M. Kany, Prof. M. El Gendy and Dr. A. El Gendy. After the death of Prof. Kany, Prof. M. El Gendy and Dr. A. El Gendy further developed the program to meet the needs of practice.

The consolidation problems outlined in this book can be also analyzed by the program *ELPLA* and the same results can be obtained. *GEO Tools* is a simple user interface program and needs little information to define a problem. It is prefer to use it for a simple foundation geometry. Furthermore, *ELPLA* can also read data files of a consolidation problem defined by *GEO Tools*. With some modifications for these data user can analyze the problem again by *ELPLA*.

4 Consolidation Settlement

4.1 Introduction

This chapter describes the essential equations to obtain the nonlinear stress distribution in the soil. Consequently, the consolidation settlement of different shapes of loadings on consolidated clay deposits can be estimated. The solution depends on closed-form equations for determining the stress and consolidation settlement in the soil.

4.2 Average vertical stress in a soil layer

The average vertical increase of stress in a layer is defined as the equivalent uniform vertical stress acting in that layer due to the applied pressure at the surface. This stress is required to carry out the procedure of consolidation settlement calculation. The essential equations of the average increase of stress in a soil layer can be derived by using the integration as described in the following paragraphs.

4.2.1 Average vertical stress due to a concentrated load

Figure 4.1 shows a concentrated load Q [kN] acting on the surface at a point *j*. According to *Boussinesq* (1885) the vertical stress σ_z [kN/m^2] on a soil element dz located at a depth z [m] under the point *i* due to the concentrated load is given by:

$$\sigma_z = \frac{3Q\,z^3}{2\pi}\left(\frac{1}{z^2+r^2}\right)^{5/2} \tag{4.1}$$

where r is the radial distance between points *i* and *j*, [m].

The average vertical stress in a soil layer of thickness h [m] due to a concentrated load at the surface can be obtained from:

$$\Delta\sigma_{va} = \frac{1}{h}\int_{h_1}^{h_2} \sigma_z \, dz \tag{4.2}$$

Substituting Eq. (4.1) in Eq. (4.2) and carrying out the integration:

$$\Delta\sigma_{va} = \frac{3Q}{2\pi h}\int_{h_1}^{h_2} z^3\left(\frac{1}{z^2+r^2}\right)^{5/2} dz$$

(4.3)

$$\Delta\sigma_{va} = \frac{Q}{2\pi\,h}\left(\frac{r^2}{\left(h_2^2+r^2\right)^{\frac{3}{2}}} - \frac{3}{\sqrt{h_2^2+r^2}} - \frac{r^2}{\left(h_1^2+r^2\right)^{\frac{3}{2}}} + \frac{3}{\sqrt{h_1^2+r^2}}\right)$$

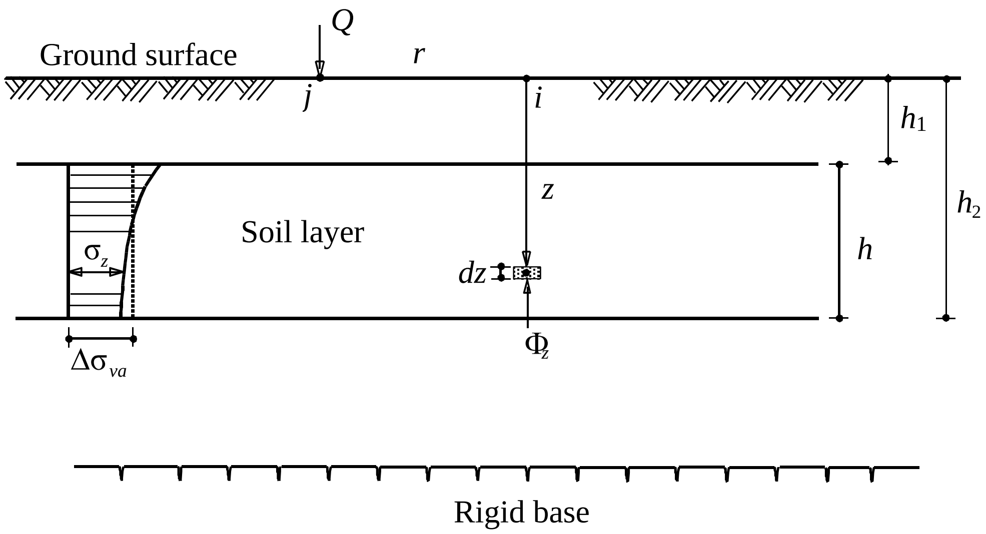


Figure 4.1 Average vertical stresses $\Delta\sigma_{va}$ in a soil layer h due to a concentrated load Q

4.2.2 Average vertical stress due to a circular loaded area

Figure 4.2 shows a circular loaded area of intensity q [kN/m^2] and a radius a [m] acting on the surface. The vertical stress σ_z [kN/m^2] on a soil element dz located at a depth z [m] under the center of the area is given by:

$$\sigma_z = q\left(1 - \frac{z^3}{\left(z^2 + a^2\right)^{3/2}}\right) \tag{4.4}$$

The average vertical stress in a soil layer of thickness h [m] due to a circular loaded area at the surface can be obtained from:

$$\Delta\sigma_{va} = \frac{1}{h}\int_{h_1}^{h_2} \sigma_z \, dz \tag{4.5}$$

Substituting Eq. (4.4) in Eq. (4.5) and carrying out the integration:

$$\Delta\sigma_{va} = \frac{q}{h}\int_{h_1}^{h_2}\left(1 - \frac{z^3}{\left(z^2 + a^2\right)^{3/2}}\right)dz$$

$$\Delta\sigma_{va} = \frac{q}{h}\left(h - \frac{h_2^2 + 2a^2}{\sqrt{h_2^2 + a^2}} + \frac{h_1^2 + 2a^2}{\sqrt{h_1^2 + a^2}}\right) \tag{4.6}$$

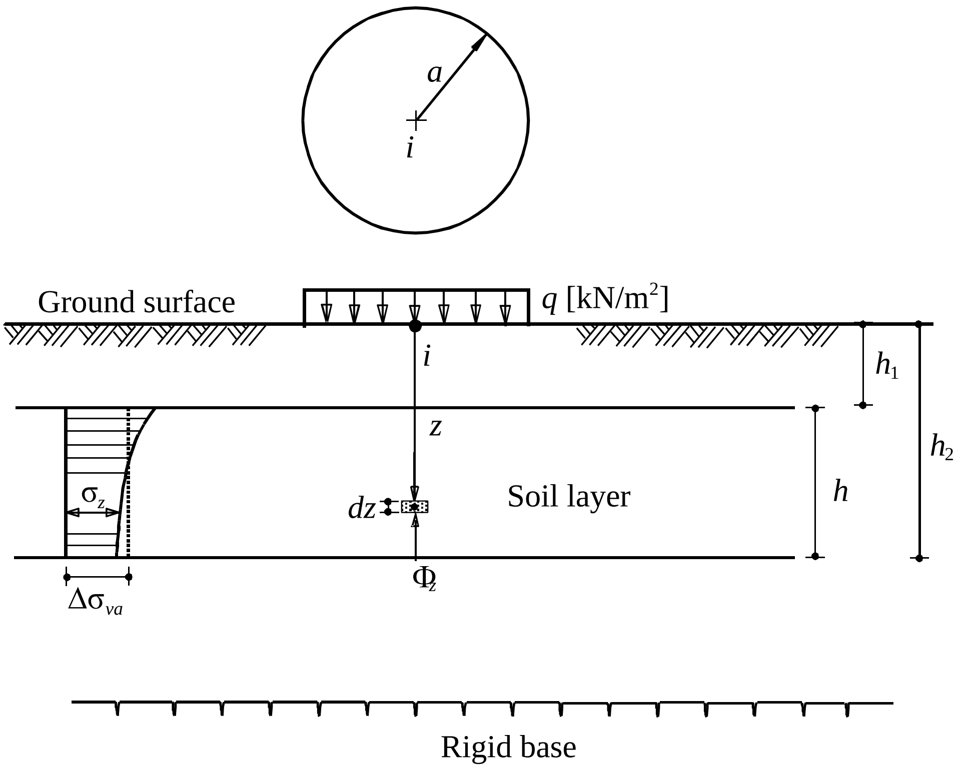


Figure 4.2 Average vertical stress $\Delta\sigma_{va}$ in a soil layer h due to a circular loaded area

4.2.3 Average vertical stress due to a triangular loaded area

Figure 4.3 shows a right triangular loaded area of intensity q [kN/m^2] with sides a [m] and b [m] acting on the surface. According to *Steinbrenner* (1934) the vertical stress σ_z [kN/m^2] on a soil element dz located at a depth z [m] under the corner i of the triangular can be expressed as:

$$\sigma_z = \frac{q}{2\pi}\left[\tan^{-1}\frac{b}{a} - \tan^{-1}\frac{b\,z}{a\sqrt{a^2+b^2+z^2}} + \frac{a\,b\,z}{\left(a^2+z^2\right)\sqrt{a^2+b^2+z^2}}\right] \tag{4.7}$$

The average vertical stress in a soil layer of thickness h [m] due to a right triangular loaded area at the surface can be obtained from:

$$\Delta\sigma_{va} = \frac{1}{h}\int_{h_1}^{h_2} \sigma_z \, dz \tag{4.8}$$

Substituting Eq. (4.7) in Eq. (4.8) and carrying out the integration:

$$\Delta\sigma_{va} = \frac{q}{2\pi h}\int_{h_1}^{h_2}\left[\tan^{-1}\frac{b}{a} - \tan^{-1}\frac{b\,z}{a\sqrt{a^2+b^2+z^2}} + \frac{a\,b\,z}{\left(a^2+z^2\right)\sqrt{a^2+b^2+z^2}}\right]dz$$

$$\Delta\sigma_{va} = \frac{q}{2\pi\, h}\left\{\left(a\ln\frac{(c_2-b)\,(m+b)}{(c_2+b)\,(m-b)} + h_2\tan^{-1}\frac{b}{a} - h_2\tan^{-1}\frac{b\,h_2}{a\,c_2}\right) - \left(a\ln\frac{(c_1-b)\,(m+b)}{(c_1+b)\,(m-b)} + h_1\tan^{-1}\frac{b}{a} - h_1\tan^{-1}\frac{b\,h_1}{a\,c_1}\right)\right\} \tag{4.9}$$

where $m=\sqrt{a^2+b^2}$, $c_1=\sqrt{a^2+b^2+h_1^2}$ and $c_2=\sqrt{a^2+b^2+h_2^2}$

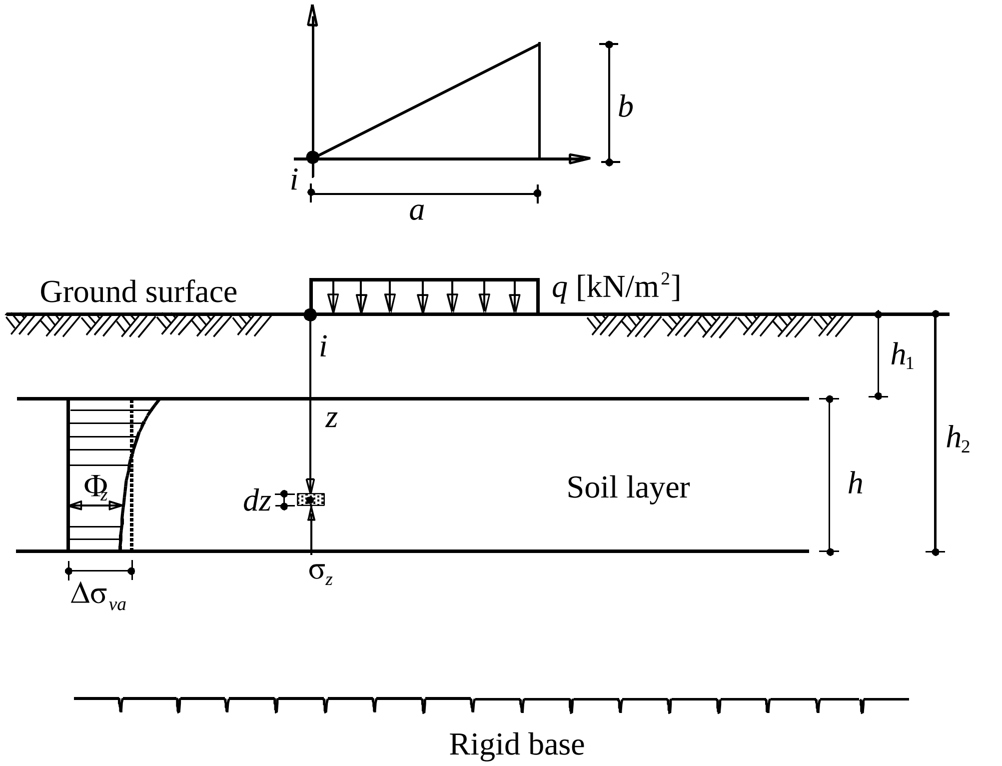


Figure 4.3 Average vertical stress $\Delta\sigma_{va}$ in a soil layer *h* due to a triangular loaded area

4.2.4 Average vertical stress due to a rectangular loaded area

Now, it is easy to create the average vertical stress due to a rectangular loaded area of intensity q [kN/m^2] with sides a [m] and b [m] acting on the surface. Thus can be done by superposing two triangular loads as shown in Figure 4.4. The average vertical stress $\Delta\sigma_{va}$ [kN/m^2] in the soil layer under the corner *i* of the rectangular can be expressed in two parts as:

$$\Delta\sigma_{va} = \Delta\sigma_{va1} + \Delta\sigma_{va2} \tag{4.10}$$

where $\Delta\sigma_{va1}$ in the average vertical stress $\Delta\sigma_{va}$ is due to triangle 1:

$$\Delta\sigma_{va1} = \frac{q}{2\pi\, h}\left\{\left(a \ln\frac{(c_2-b)(m+b)}{(c_2+b)(m-b)} + h_2 \tan^{-1}\frac{b}{a} - h_2 \tan^{-1}\frac{b\, h_2}{a\, c_2}\right) - \left(a \ln\frac{(c_1-b)(m+b)}{(c_1+b)(m-b)} + h_1 \tan^{-1}\frac{b}{a} - h_1 \tan^{-1}\frac{b\, h_1}{a\, c_1}\right)\right\} \quad (4.11)$$

where $\Delta\sigma_{va2}$ in the average vertical stress $\Delta\sigma_{va}$ is due to triangle 2:

$$\Delta\sigma_{va2} = \frac{q}{2\pi\, h}\left\{\left(b \ln\frac{(c_2-a)(m+a)}{(c_2+a)(m-a)} + h_2 \tan^{-1}\frac{a}{b} - h_2 \tan^{-1}\frac{a\, h_2}{b\, c_2}\right) - \left(b \ln\frac{(c_1-a)(m+a)}{(c_1+a)(m-a)} + h_1 \tan^{-1}\frac{a}{b} - h_1 \tan^{-1}\frac{a\, h_1}{b\, c_1}\right)\right\} \quad (4.12)$$

It can be seen that $\Delta\sigma_{va1}$ and $\Delta\sigma_{va2}$ have the roles of *a* and *b* reversed to account for the position of the corner *i* relative to the two sides of the appropriate triangle.

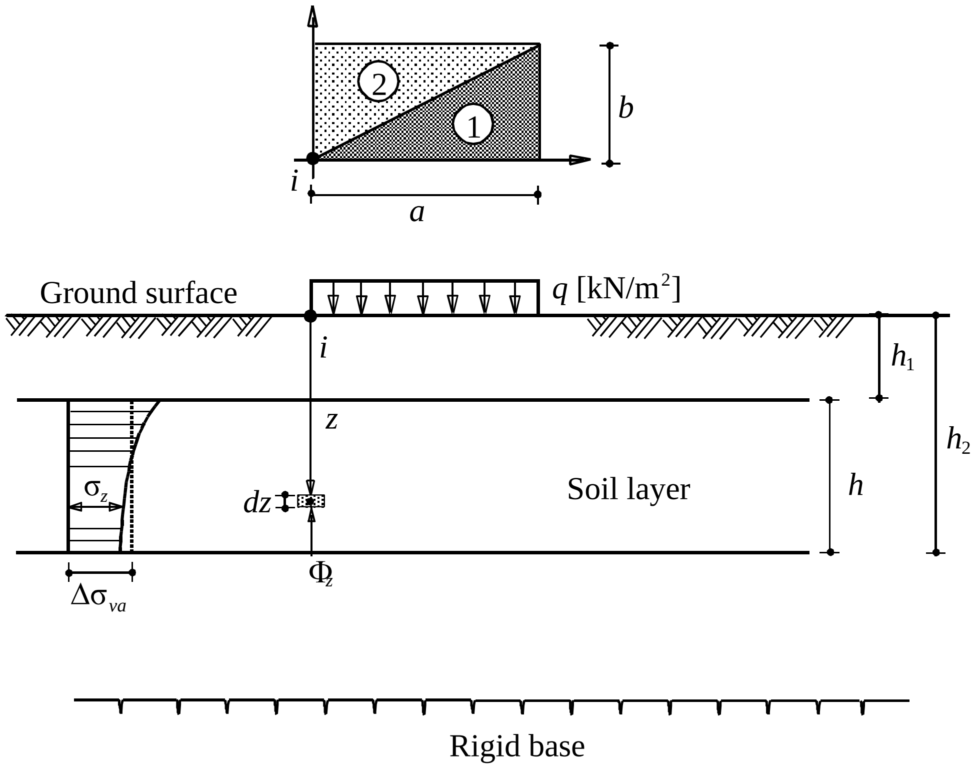


Figure 4.4 Average vertical stress $\Delta\sigma_{va}$ in a soil layer *h* due to a rectangular loaded area

Substituting Eqns. (4.11) and (4.12) in Eq. (4.10) yields:

$$\Delta\sigma_{va} = \frac{q}{2\pi h}\left\{\left(b \ln\frac{(c_2-a)(m+a)}{(c_2+a)(m-a)} + a \ln\frac{(c_2-b)(m+b)}{(c_2+b)(m-b)} + h_2 \tan^{-1}\frac{ab}{h_2 c_2}\right)\right.$$
$$\left. - \left(b \ln\frac{(c_1-a)(m+a)}{(c_1+a)(m-a)} + a \ln\frac{(c_1-b)(m+b)}{(c_1+b)(m-b)} + h_1 \tan^{-1}\frac{ab}{h_1 c_1}\right)\right\} \quad (4.13)$$

4.3 Consolidation settlement when soil properties of the clay layer is defined by C_c

The consolidation settlement calculated by C_c is more realistic. Because this calculation takes into account the nonlinear soil stresses in the soil layers due to the applied pressure at the surface. In this chapter a procedure using integration is developed to determine the consolidation settlement under arbitrary shapes of loading such as concentrated load, circular loaded area, triangular loaded area and rectangular loaded area.

4.3.1 Consolidation for a thin clay layer

Sometimes the clay layer is thin relative to the raft dimensions. In this case it will be sufficiently accurate to consider the increase of vertical stress at mid-depth of the layer or the average vertical stress in the entire layer. The consolidation settlement S_c [m] in a layer of a thickness h [m], Figure 4.5, can be obtained from the following equations whichever is applicable:

For a normally consolidated clay, if $\sigma_o \exists \sigma_c$ (loading case), Figure 4.6:

$$S_c = \frac{C_c\, h}{1+e_o}\log\left(\frac{\sigma_o + \Delta\sigma_{av}}{\sigma_o}\right) \quad (4.14)$$

For a preconsolidated clay, if $\sigma_f < \sigma_c$ (reloading case), Figure 4.7:

$$S_c = \frac{C_r\, h}{1+e_o}\log\left(\frac{\sigma_o + \Delta\sigma_{av}}{\sigma_o}\right) \quad (4.15)$$

For a preconsolidated clay, if $\sigma_f > \sigma_c > \sigma_o$ (loading and reloading case), Figure 4.8:

$$S_c = \frac{C_r\, h}{1+e_o}\log\left(\frac{\sigma_c}{\sigma_o}\right) + \frac{C_c\, h}{1+e_o}\log\left(\frac{\sigma_o + \Delta\sigma_{av}}{\sigma_c}\right) \quad (4.16)$$

where:

σ_o Initial overburden pressure at the middle of the layer, $\sigma_o = \gamma z$, [kN/m^2].
$\Delta\sigma_{av}$ Average increase of stress in the layer due to the applied load, [kN/m^2].
σ_c Preconsolidation pressure of the layer, [kN/m^2].
C_c Compression index for loading, [-].
C_r Compression index for reloading, [-].
e_o Initial void ratio, [-].
e_f Final void ratio, [-].

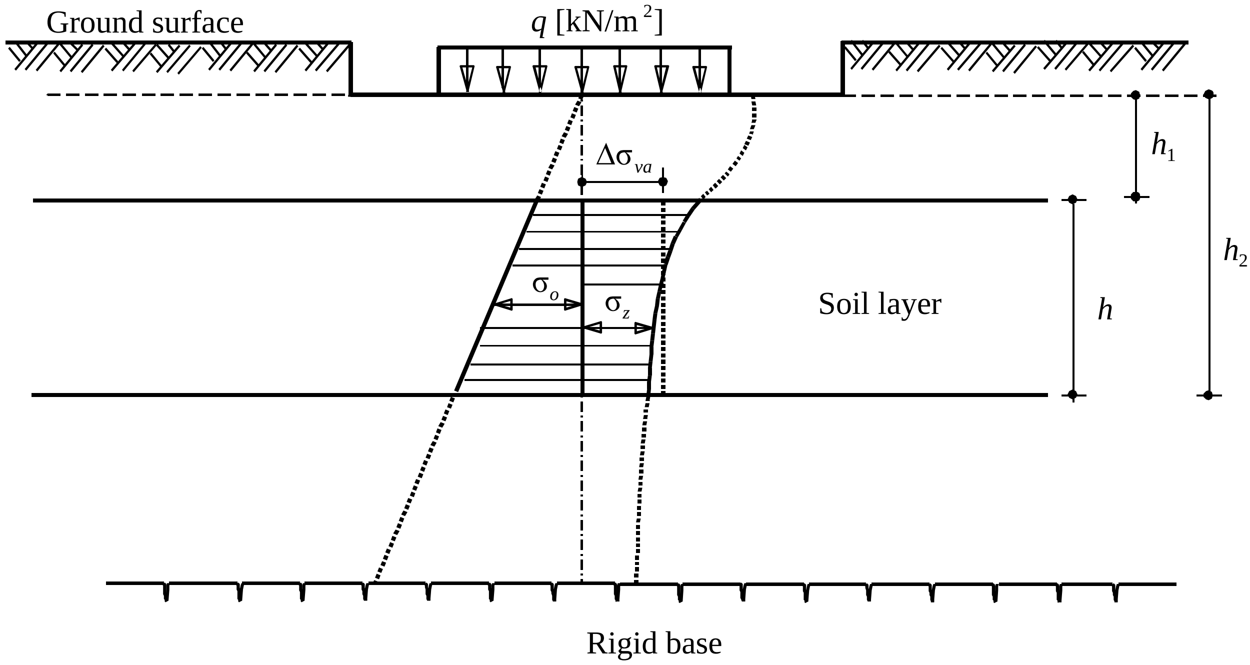


Figure 4.5 Calculation of consolidation settlement

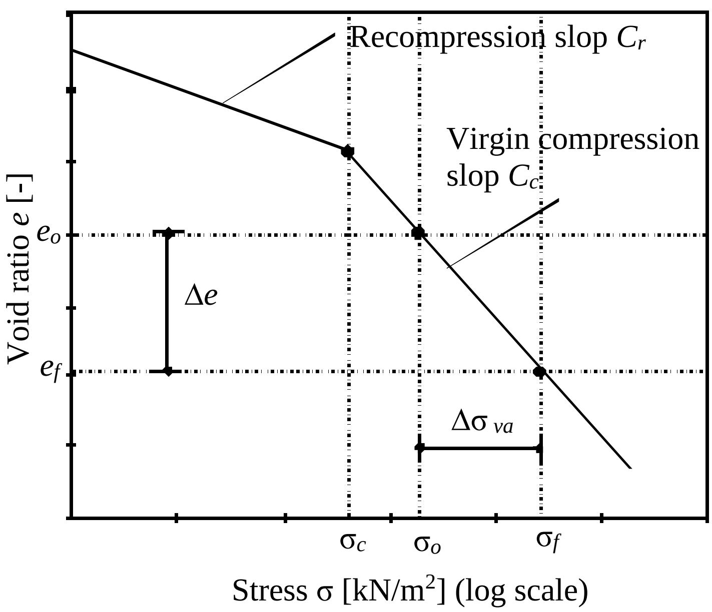


Figure 4.6 Relationship between void ratio e and stress σ ($\sigma_o \exists \sigma_c$) (loading case)

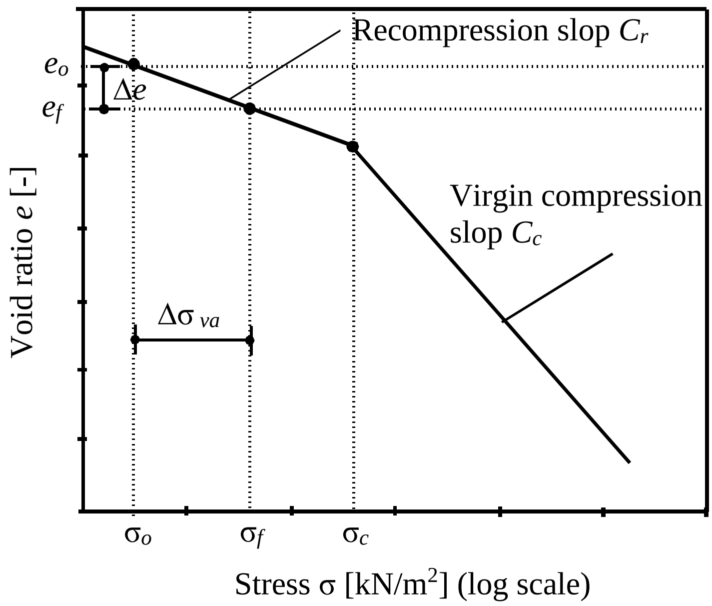


Figure 4.7 Relationship between void ratio *e* and stress σ ($\sigma_f < \sigma_c$) (reloading case)

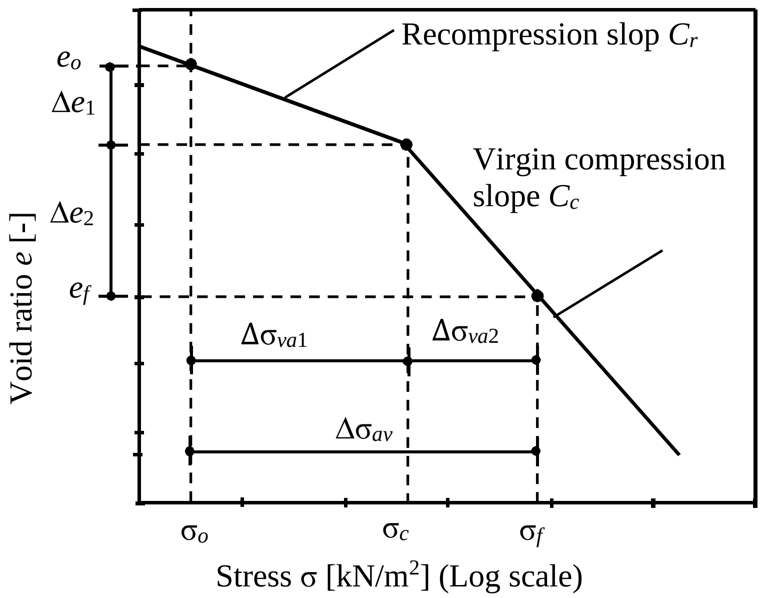


Figure 4.8 Relationship between void ratio *e* and stress σ ($\sigma_f > \sigma_c > \sigma_o$) (loading and reloading case)

4.3.2 Consolidation for a thick clay layer

When a load is applied over a limited area, the increase of pressure in the soil due to the applied load will decrease with the depth as shown in Figure 4.9. For more realistic consolidation settlement predication, the calculation of consolidation settlement of a thick layer can be carried out by dividing the given layer into *n* sub-layers as shown in Figure 4.9. Then, determining the average increase of stress $\Delta\sigma_{av}$ in each sub-layer due to the applied load. Consequently, the consolidation settlement can be determined due to this increase of stress in each sub-layer. *El Gendy* (2003) had presented a guideline for the thickness of the sub-layer depending on the foundation dimension and load. The total consolidation settlement for the entire clay layer will be the summation of the consolidation settlements for all individual sub-layers.

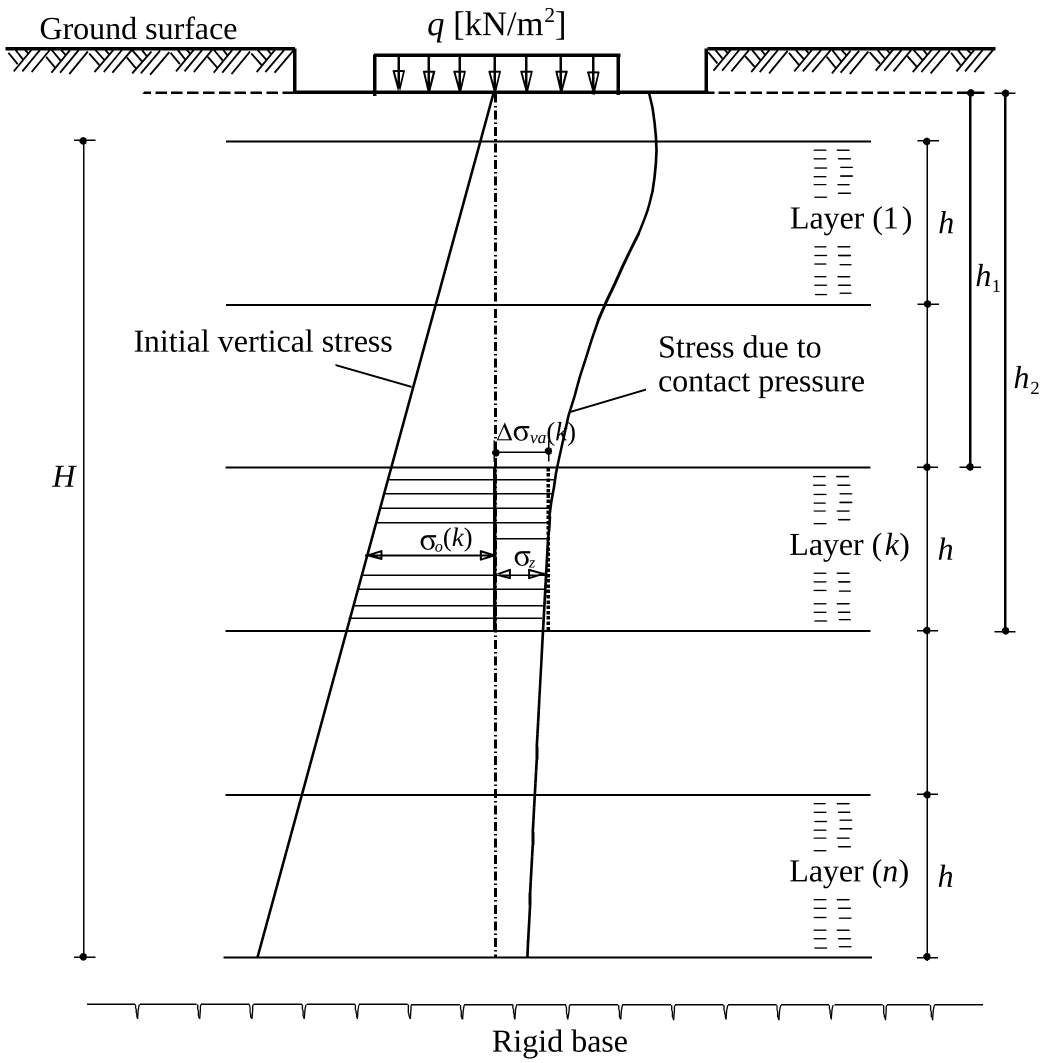


Figure 4.9 Variation of vertical stress σ_z with depth

4.3.3 Over consolidated ratio OCR

The nonlinear analysis use the preconsolidation pressure of the layer σ_c to determine if the soil is over consolidated or normally consolidated. Instead of specifying σ_c for the entire layer, it may instead specify the over consolidation ratio *OCR*, where:

$$OCR = \frac{\sigma_c}{\sigma_o} \tag{4.17}$$

Two cases concerning *OCR* are considered as follows:

OCR = 1: The clay is normally consolidated and the settlement is determined from Eq. (4.14).

OCR > 1: The clay is over consolidated and the settlement is determined from Eq. (4.16).

4.4 Consolidation settlement when soil properties of the clay layer is defined by m_v

Clay properties may be represented by either the coefficient of volume change m_v (volume change is the inverse of modulus of compressibility, $m_v = 1/E_s$) or compression index C_c. The calculation procedures of consolidation settlement which its soil properties are represented by E_s may be considered also as an alternative simple solution to estimate the consolidation settlement.

The consolidation settlement S_c [m] in a layer of thickness h [m] is given by:

$$S_c = m_v \Delta\sigma_{va}\, h = \frac{1}{E_s} \Delta\sigma_{va}\, h \tag{4.18}$$

where:

$\Delta\sigma_{av}$	Average increase of stress in the layer due to the applied load, [kN/m^2]
$m_v = 1/E_s$	Coefficient of volume change of the layer, [m^2/ kN]
E_s	Modulus of compressibility of the layer, [kN/m^2]
h	Layer thickness, [m]

4.4.1 Consolidation settlement due to a concentrated load

The consolidation settlement S_c [m] in a soil layer of a thickness h [m] due to a concentrated load at the surface can be given by substituting Eq. (4.3) in Eq. (4.17):

$$S_c = \frac{Q}{2\pi E_s}\left(\frac{r^2}{\left(h_2^2 + r^2\right)^{\frac{3}{2}}} - \frac{3}{\sqrt{h_2^2 + r^2}} - \frac{r^2}{\left(h_1^2 + r^2\right)^{\frac{3}{2}}} + \frac{3}{\sqrt{h_1^2 + r^2}} \right) \tag{4.19}$$

The consolidation settlement at the surface $S_c(0)$ in an infinite soil layer for isotropic elastic half-space medium is obtained by putting $h_1 = 0$ and $h_2 = 4$ in Eq. (4.18):

$$S_c(0) = \frac{Q}{\pi E_s r} \tag{4.20}$$

The consolidation settlement $S_c(z)$ at any depth z in an infinite soil layer for isotropic elastic half-space medium is obtained by putting $h_1 = z$ and $h_2 = 4$ in Eq. (4.18):

$$S_c(z) = \frac{Q}{2\pi\, E_s}\left(\frac{3}{\sqrt{z^2 + r^2}} - \frac{r^2}{\left(z^2 + r^2\right)^{\frac{3}{2}}} \right) \tag{4.21}$$

4.4.2 Consolidation settlement due to a circular loaded area

The consolidation settlement S_c [m] in a soil layer of a thickness h [m] due to a circular loaded area at the surface can be given by substituting Eq. (4.6) in Eq. (4.17):

$$S_c = \frac{q}{E_s}\left(h - \frac{h_2^2 + 2a^2}{\sqrt{h_2^2 + a^2}} + \frac{h_1^2 + 2a^2}{\sqrt{h_1^2 + a^2}} \right) \tag{4.22}$$

The consolidation settlement at the surface $S_c(0)$ in an infinite soil layer for isotropic elastic half-space medium is obtained by putting $h_1 = 0$ and $h_2 = 4$ in Eq. (4.21):

$$S_c(0) = \frac{2qa}{E_s} \tag{4.23}$$

The consolidation settlement $S_c(z)$ at any depth z in an infinite soil layer for isotropic elastic half-space medium is obtained by putting $h_1 = z$ and $h_2 = 4$ in Eq. (4.21):

$$S_c(z) = \frac{q}{E_s}\left(\frac{z^2 + 2a^2}{\sqrt{z^2 + a^2}} - z \right) \tag{4.24}$$

4.4.3 Consolidation settlement due to a triangular loaded area

The consolidation settlement S_c [m] in a soil layer of a thickness h [m] due to a right triangular loaded area at the surface can be given by substituting Eq. (4.9) in Eq. (4.17):

$$\begin{aligned} S_c = \frac{q}{2\pi\, E_s}\Bigg\{ &\left(a \ln \frac{(c_2 - b)\,(m+b)}{(c_2 + b)\,(m-b)} + h_2 \tan^{-1}\frac{b}{a} - h_2 \tan^{-1}\frac{b\, h_2}{a\, c_2} \right) \\ &- \left(a \ln \frac{(c_1 - b)\,(m+b)}{(c_1 + b)\,(m-b)} + h_1 \tan^{-1}\frac{b}{a} - h_1 \tan^{-1}\frac{b\, h_1}{a\, c_1} \right) \Bigg\} \end{aligned} \tag{4.25}$$

The consolidation settlement at the surface $S_c(0)$ in an infinite soil layer for isotropic elastic half-space medium is obtained by putting $h_1 = 0$ and $h_2 = 4$ in Eq. (4.24):

$$S_c(0) = \frac{q}{2\pi\, E_s}\left(a \ln \frac{(m+b)}{(m-b)} \right) \tag{4.26}$$

The consolidation settlement $S_c(z)$ at any depth z in an infinite soil layer for isotropic elastic half-space medium is obtained by putting $h_1 = z$ and $h_2 = 4$ in Eq. (4.24):

$$S_c(z) = \frac{q}{2\pi E_s}\left(a \ln \frac{(c-b)(m+b)}{(c+b)(m-b)} + z \tan^{-1}\frac{b}{a} - z \tan^{-1}\frac{b\,z}{a\,c}\right) \tag{4.27}$$

where $c = \sqrt{a^2 + b^2 + z^2}$

4.4.4 Consolidation settlement due to a rectangular loaded area

The consolidation settlement S_c [m] in a soil layer of a thickness h [m] due to a rectangular loaded area at the surface can be given by substituting Eq. (4.13) in Eq. (4.17):

$$S_c = \frac{q}{2\pi E_s}\left\{\left(b \ln \frac{(c_2-a)(m+a)}{(c_2+a)(m-a)} + a \ln \frac{(c_2-b)(m+b)}{(c_2+b)(m-b)} + h_2 \tan^{-1}\frac{ab}{h_2 c_2}\right) - \left(b \ln \frac{(c_1-a)(m+a)}{(c_1+a)(m-a)} + a \ln \frac{(c_1-b)(m+b)}{(c_1+b)(m-b)} + h_1 \tan^{-1}\frac{ab}{h_1 c_1}\right)\right\} \tag{4.28}$$

The consolidation settlement at the surface $S_c(0)$ in an infinite soil layer for isotropic elastic half-space medium is obtained by putting $h_1 = 0$ and $h_2 = 4$ in Eq. (4.27):

$$S_c(0) = \frac{q}{2\pi E_s}\left(b \ln \frac{(m+a)}{(m-a)} + a \ln \frac{(m+b)}{(m-b)}\right) \tag{4.29}$$

The consolidation settlement $S_c(z)$ at any depth z in an infinite soil layer for isotropic elastic half-space medium is obtained by putting $h_1 = z$ and $h_2 = 4$ in Eq. (4.27):

$$S_c(z) = \frac{q}{2\pi E_s}\left(b \ln \frac{(c-a)(m+a)}{(c+a)(m-a)} + a \ln \frac{(c-b)(m+b)}{(c+b)(m-b)} + z \tan^{-1}\frac{ab}{z\,c}\right) \tag{4.30}$$

4.5 Settlement at any corner of a non-right-triangular loaded area

In the finite element method, the foundation is represented by a series of either rectangular or triangular elements. In such a case influence factors for settlements under the nodes are required. So far it has considered the settlement under a corner of loaded area either a right triangle or a rectangle. The settlement of a right triangle can be generalized to find the settlement at any other point of any triangular-shaped loaded area using the principle of superposition. The settlement at any corner of any triangular-shaped loaded area is given by the settlements associated with two right triangles. For a corner i of a general triangle $\Delta\ ijk$ of sides T_i, T_j and T_k, Figure 4.10, the consolidation settlement S_c can be obtained using the principle of superposition by the following general Eq. (4.30):

$$S_c = S_c(\zeta,\eta) + S_c(\zeta,\xi) \tag{4.31}$$

where $\eta = T_k \cos(j), \xi = T_j \cos(k), \zeta = \sqrt{T_k^2 - \eta^2}$ and

It can be noticed that adding or subtracting the associated settlements $S_c(\zeta,\ \eta)$ and $S_c(\zeta,\ \xi)$ depends on the singe of η and ξ, where the values of η and ξ may be either positive or negative with regard to the sign of cos (j) and cos (k). It can be seen from settlement Eqns (4.24) to (4.26) that a negative value of η or ξ leads to a negative contribution of the associated settlement, while η=0 or ξ=0 eliminate the contribution of the associated settlement.

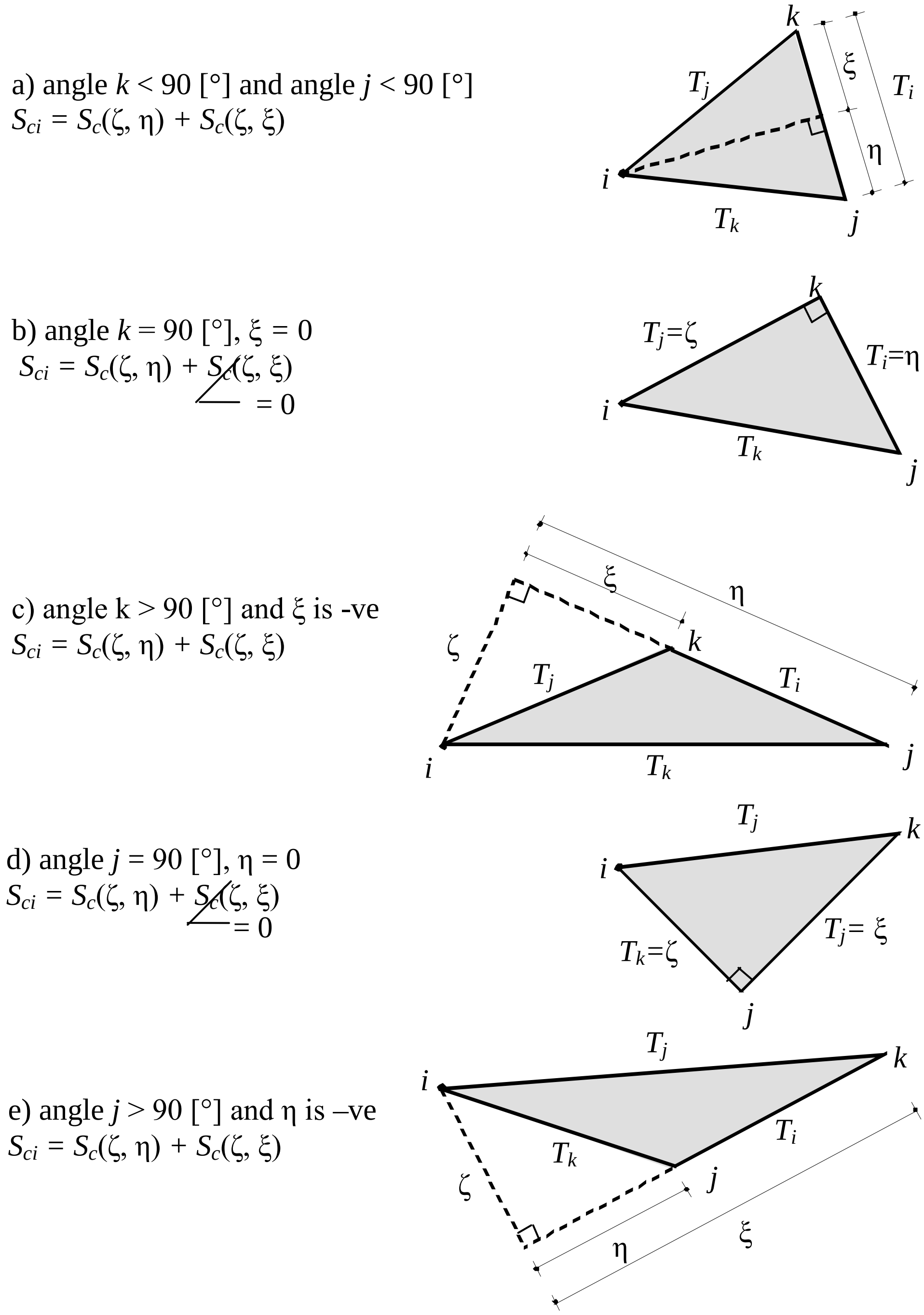


Figure 4.10 Superposition principle to find the settlement at any corner i of any triangular-shaped loaded area

4.6 Settlement at a point outside the corners of a non-right-triangular loaded area

Adding and subtracting corner settlements for three triangular loaded areas can obtain the settlement of any point outside the corners of any triangular-shaped loaded area as shown in Figure 4.11. For a point *o* outside the corners of a general triangle Δijk, Figure 4.11, the consolidation settlement S_c can be obtained using the principle of superposition by the following general Eq. (4.31):

$$S_{co} = S_c(\Delta oij) + S_c(\Delta ojk) + S_c(\Delta oki) \qquad (4.32)$$

where:

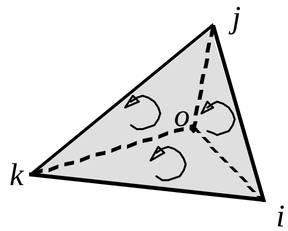
$S_c(\Delta oij)$ Settlement at point *o* due to the triangular loaded area Δoij, [m]
$S_c(\Delta ojk)$ Settlement at point *o* due to the triangular loaded area Δojk, [m]
$S_c(\Delta oki)$ Settlement at point *o* due to the triangular loaded area Δoki, [m]

To apply Eq. (4.31) for any triangular-shaped loaded area the numbering of the triangle Δijk must be counterclockwise. It can be noticed that the sign of the associated settlement $S_c(\Delta oij)$, $S_c(\Delta ojk)$ or $S_c(\Delta oki)$ depends on the counter of the triangle where the sign is positive when the triangle is counterclockwise and vice verse. As example, in Figure 4.11b the triangles Δoij and Δojk give a positive contribution to the settlement S_c at point *o* while the triangle Δoki contributes negatively. The sign of the associated settlements may be determined from the following Eq. (4.32):

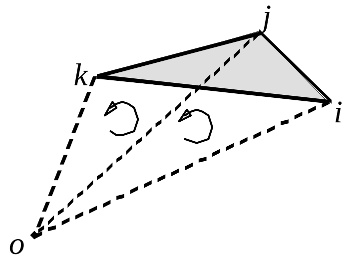
$$g = x_l\, y_{l+1} - x_{l+1}\, y_l + x_{l+1}\, y_K - x_{l+2}\, y_{l+1} + x_{l+2}\, y_l - x_l\, y_{l+2} \qquad (4.33)$$

where x_l and y_l are the corner coordinates of the examined triangle, l corner numbering and g is a parameter indicates the sign of the associated settlements. The sign of the parameter value is the same as that of the associated settlement.

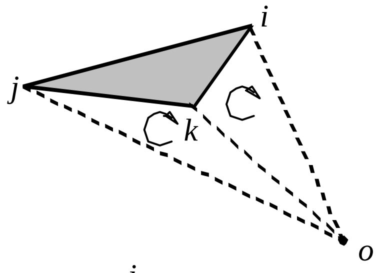
a) Point *o* lies inside the triangle
$S_{co} = S_c(\Delta oij) + S_c(\Delta ojk) + S_c(\Delta oki)$



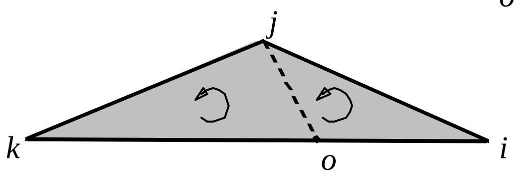
b) Point *o* lies outside the triangle
$S_{co} = S_c(\Delta oij) + S_c(\Delta ojk) + S_c(\Delta oki)$



c) Point *o* lies outside the triangle
$S_{co} = S_c(\Delta oij) + S_c(\Delta ojk) + S_c(\Delta oki)$



d) Point *o* lies at a side of the triangle
$S_{co} = S_c(\Delta oij) + S_c(\Delta ojk) + \underbrace{S_c(\Delta oki)}_{= 0}$



e) Point *o* lies at a coincident line with a side of the triangle
$S_{co} = S_c(\Delta oij) + S_c(\Delta ojk) + \underbrace{S_c(\Delta oki)}_{= 0}$

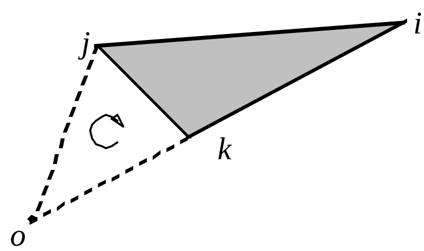
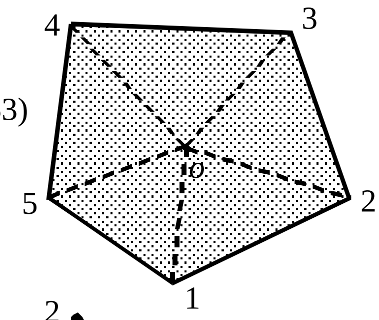


Figure 4.11 Superposition principle to find the settlement at any point *o* outside the corners of any triangular-shaped loaded area

4.7 Settlement at a point inside or outside any polygon loaded area

Considering the above rules described in cases (a) to (e) in Figure 4.10 and Figure 4.11, the settlement for any polygon loaded are may be obtained as indicated in Figure 4.12 according to *Damy*/ *Casales* (1981).

a) Settlement for interior node
$S_{co} = S_c(o12)+S_c(o23)+S_c(o34)+S_c(o45)+S_c(o53)$



b) Settlement for exterior node
$S_{co} = S_c(o12)+S_c(o23)+S_c(o34)+S_c(o54)+S_c(o15)$

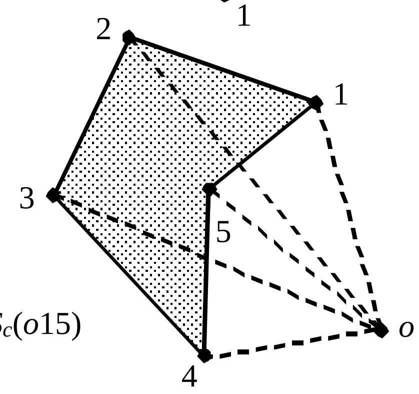


Figure 4.12 Stress coefficient for polygonal loaded area

4.8 Soil properties and parameters

4.8.1 *Poisson*'s ratio ν_s

Poisson's ratio ν_s for a soil is defined as the ratio of lateral strain to longitudinal strain. It can be evaluated from the Triaxial test. *Poisson*'s ratio ν_s can be determined from at-rest earth pressure coefficient K_o as follows

$$\nu_s = \frac{K_o}{1 + K_o} \tag{8.34}$$

Some typical values for *Poisson*'s ratio are shown in Table 4.1 according to *Bowles* (1977). *Poisson*'s ratio in general ranges between 0 and 0.5.

Table 4.1 Typical range of values for *Poisson*'s ratio ν_s according to *Bowles* (1977)

Type of soil	*Poisson*'s ratio ν_s [-]
Clay, saturated	0.4 - 0.5
Clay, unsaturated	0.1 - 0.3
Sandy clay	0.2 - 0.3
Silt	0.3 - 0.35
Sand, dense	0.2 - 0.4
Sand, coarse (void ratio = 0.4 - 0.7)	0.15
Sand, fine grained (void ratio = 0.4 - 0.7)	0.25
Rock	0.1 - 0.4

4.8.2 Moduli of compressibility E_s and W_s and unit weight of the soil γ_s

The equations derived in the previous section for calculation of flexibility coefficients require either the moduli of compressibility for loading E_s and reloading W_s or moduli of elasticity for loading E and reloading W for the soil. The yielding of the soil is described by these elastic moduli. The moduli of compressibility E_s and W_s can be determined from the stress-strain curve through a confined compression test (for example Odometer test) as shown in Figure 4.13. In this case, the deformation will occur in the vertical direction only. Therefore, if the moduli of compressibility E_s and W_s are determined from a confined compression test, *Poisson*'s ratio will be taken $\nu_s = 0.0$. If the other moduli of elasticity E and W are used in the equations derived in the previous section, *Poisson*'s ratio will be taken to be $\nu_s \neq 0$. In general, *Poisson*'s ratio ranges in the limits $0 < \nu_s < 0.5$.

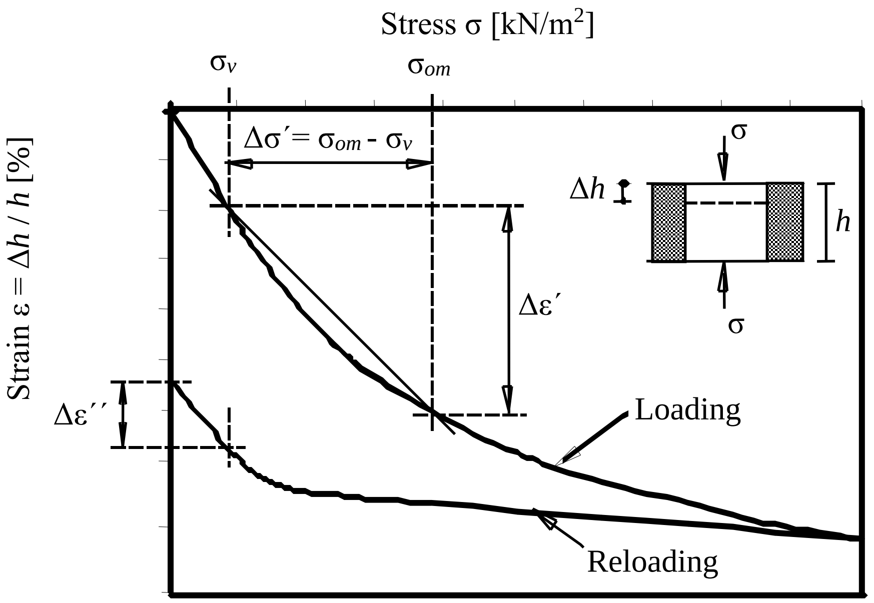


Figure 4.13 Stress-strain diagram from confined compression test (Oedometer test)

The modulus of compressibility E_s [kN/m^2] (or W_s [kN/m^2]) is defined as the ratio of the increase in stress Δσ to decrease in strain Δε as (Figure 4.13)

$$\left.\begin{aligned} E_s &= \frac{\Delta\sigma'}{\Delta\varepsilon'} = \frac{\sigma_{om}\text{-}\sigma_v}{\Delta\varepsilon'} \\ W_s &= \frac{\Delta\sigma''}{\Delta\varepsilon''} = \frac{\sigma_v}{\Delta\varepsilon''} \end{aligned}\right\} \tag{4.35}$$

where:

$\Delta\sigma'$	Increase in stress from σ_v to σ_{om}	[kN/m^2]
σ_v	Stress equal to overburden pressure	[kN/m^2]
σ_{om}	Stress equal to expected average stress on the soil	[kN/m^2]
$\Delta\varepsilon'$	Decrease in strain due to stress from σ_v to σ_{om}	[-]
$\Delta\sigma''$	Increase in stress due to reloading	[kN/m^2]
$\Delta\varepsilon''$	Decrease in strain due to reloading	[-]

The moduli of compressibility may be expressed in terms of either void ratio or specimen thickness. For an increase in effective stress Δσ to decrease in void ratio Δ*e*, the moduli of compressibility E_s [kN/m^2] and W_s [kN/m^2] are then expressed as

$$\left.\begin{aligned} E_s &= \frac{1}{m_v'} = \frac{\Delta\sigma'(1+e_o')}{\Delta e'} \\ W_s &= \frac{1}{m_v''} = \frac{\Delta\sigma''(1+e_o'')}{\Delta e''} \end{aligned}\right\} \tag{4.36}$$

where:

m'_v	Coefficient of volume change for loading	[m^2/kN]
m''_v	Coefficient of volume change for reloading	[m^2/kN]
e'_o	Initial void ratio for loading	[-]
e''_o	Initial void ratio for reloading	[-]
$\Delta e'$	Decrease in void ratio due to loading	[-]
$\Delta e''$	Decrease in void ratio due to reloading	[-]

The values of E_s and W_s for a particular soil are not constant but depend on the stress range over which they are calculated. Therefore, for linear analysis it is recommended to determine the modulus of compressibility for loading E_s at the stress range from σ_v to σ_{om}, while that for reloading W_s for a stress increment equal to the overburden pressure σ_v. On the other hand, since the modulus of compressibility increases with the depth of the soil, for more accurate analysis the modulus of compressibility may be taken increasing linearly with depth. Also, according to *Kany* (1976) the moduli of compressibility E_s and W_s may be taken depending on the stress on soil. In these two cases, the moduli of compressibility E_s and W_s can be defined in the analysis for several sub-layers instead of one layer of constants E_s and W_s.

As a rule, before the analysis the soil properties are defined through the tests of soil mechanics, particularly the moduli of compressibility E_s and W_s. For precalculations Table 8.2 for specification of the modulus of compressibility E_s can also be used.

According to *Kany* (1974), the values of W_s range between 3 to 10 times of E_s. From experience, the modulus of compressibility W_s for reloading can be taken 1.5 to 5 times as the modulus of compressibility E_s for loading.

For geologically strongly preloaded soil, the calculation is often carried out only with the modulus of compressibility for reloading W_s. In this case, the same values are defined for E_s and W_s.

Matching with the reality, satisfactory calculations of the settlements are to be expected only if the soil properties are determined exactly from the soil mechanical laboratory, field tests or back calculation of settlement measurements.

Table 4.2 shows mean moduli of compressibility E_s and the unit weight of the soil γ_s for various types of soil according to EAU (1990).

Table 4.2 Mean moduli of compressibility E_s and the unit weight of the soil γ_s for various types of soil

Type of soil	Unit weight γ_s [kN/m³]		Modulus of compressibility E_s [kN/m²]
	above water	under water	
Non-cohesive soil			
Sand, loose, round	18	10	20000 - 50000
Sand, loose, angular	18	10	40000 - 80000
Sand, medium dense, round	19	11	50000 - 100000
Sand, medium dense, angular	19	11	80000 - 150000
Gravel without sand	16	10	100000 - 200000
Coarse gravel, sharp edge	18	11	150000 - 300000
Cohesive soil			
Clay, semi-firm	19	9	5000 - 10000
Clay, stiff	18	8	2500 - 5000
Clay, soft	17	7	1000 - 2500
Boulder clay, solid	22	12	30000 - 100000
Loam, semi-firm	21	11	5000 - 20000
Loam, soft	19	9	4000 - 8000
Silt	18	8	3000 - 10000

4.8.3 Moduli of elasticity *E* and *W*

The equations derived in the previous section to determine the flexibility coefficients are used with moduli of elasticity E and W for unconfined lateral strain with *Poisson*'s ratio $\nu_s \neq 0$. It must be pointed out that, when defining *Poisson*'s ratio by $\nu_s = 0$ (limit case), the moduli of compressibility E_s and W_s for confined lateral strain (for example from Odometer test) also can be used.

The modulus of elasticity is often determined from an unconfined Triaxial compression test, Figure 4.14. Plate loading tests may also be used to determine the in situ modulus of elasticity of the soil as elastic and isotropic.

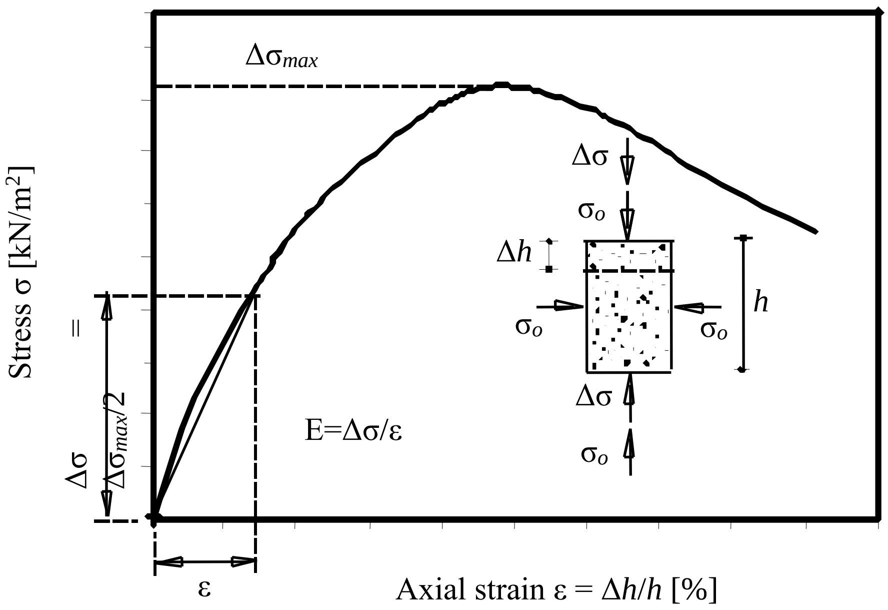


Figure 4.14 Modulus of elasticity *E* from Triaxial test

It is possible to obtain an expression for the moduli of elasticity *E* and *W* in terms of moduli of compressibility E_s, W_s and *Poisson*'s ratio ν_s for the soil as

$$\left.\begin{aligned} E &= E_s\,\frac{1-\nu_s-2\nu_s^2}{1-\nu_s} \\ W &= W_s\,\frac{1-\nu_s-2\nu_s^2}{1-\nu_s} \end{aligned}\right\} \tag{4.37}$$

The above equation shows that:

- In the limit case $\nu_s = 0$ (deformation without lateral strain), the values of *E* and E_s (also *W* and W_s) are equal

- In the other limit case $\nu_s = 0.5$ (deformation with constant volume), the moduli of elasticity will be $E = 0 \times E_s$ and $W = 0 \times W_s$. In this case, only the immediate settlement (lateral deformation with constant volume) can be determined.

Table 4.3 shows some typical values of modulus of elasticity according to *Bowles* (1977).

Table 4.3 Typical range of moduli of elasticity E for selected soils

Type of soil	Modulus of elasticity E [kN/m^2]
Very soft clay	3000 - 3000
Soft clay	2000 - 4000
Medium clay	4500 - 9000
Hard clay	7000 - 20000
Sandy clay	30000 - 42500
Silt	2000 - 20000
Silty sand	5000 - 20000
Loose sand	10000 - 25000
Dense sand	50000 - 100000
Dense sand and gravel	80000 - 200000
Loose sand and gravel	50000 - 140000
Shale	140000 - 1400000

4.8.4 Compression index C_r und initial void ratio e_o

In case of clayey soil it is recommended to use the settlement parameters C_c, C_r and C_s to represent the elastic properties of the soil in the computation of consolidation settlements. These parameters or indices can be obtained directly from the consolidation test or indirect using some empirical equations such as Equations 4.40 and 4.41.

4.8.5 Compression index C_c from consolidation test

The typical relationship between the void ratio e and effective stress σ obtained from the consolidation test is shown in Figure 4.15. The slope of the end part of the e versus log σ curve is denoted as the Compression index Cc and computed as

$$C_c = \frac{\Delta e}{\log \frac{\sigma_2}{\sigma_1}} \qquad (4.38)$$

By analogy, the other indices C_r and C_s can be obtained as shown in Figure 4.15 and Equation 4.38

$$C_r \text{ or } C_c = \frac{\Delta e}{\log \frac{\sigma_2}{\sigma_i}} \qquad (4.39)$$

where:

C_r	Recompression index	[-]
C_s	Swell index	[-]
Δe	Change in void ratio between σ_i and σ_2	[-]
σ_i	Any pressure along the appropriate curve	[kN/m^2]

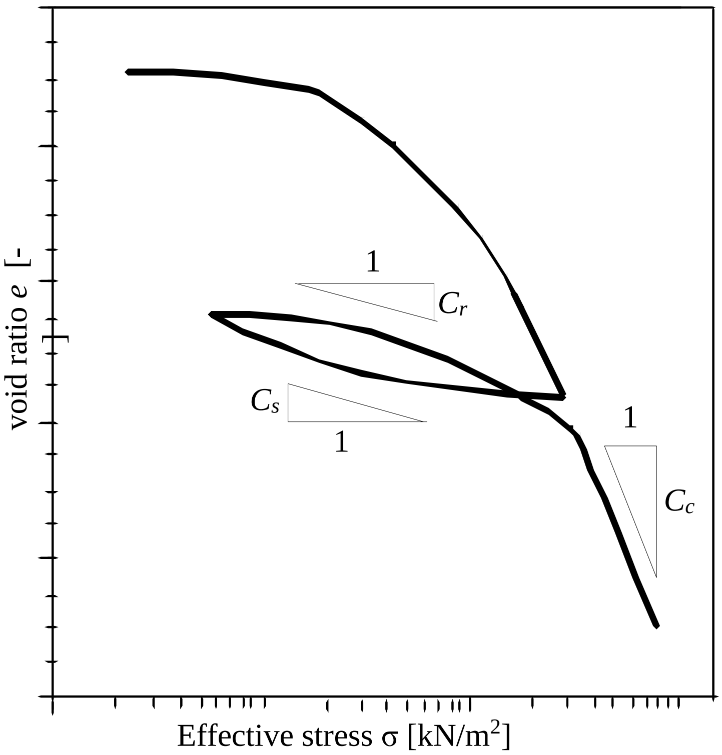


Figure 4.15 Relationship between void ratio and effective stress obtained from consolidation test

4.8.6 Compression index C_c from empirical equations

Because of the number of consolidation tests to obtain the compression indices for a given project is limited, it is often desirable to obtain approximate values by using other soil parameters which are more easily determined. Approximate values may be used for preliminary calculations or to check the laboratory data.

For normally consolidated clays *Terzaghi*/ *Peck* (1967), on the basis of research on undisturbed clays, proposed the following equation to obtain the Compression index C_c [-] from the liquid limit of the soil *LL* [%]

$$C_c = 0.009\,(LL - 10) \tag{4.40}$$

Azzouz (1976) lists several equations to obtain the compression index, one of them is given below to obtain the Compression index C_c [-] from the initial void ratio e_o [-] of the soil

$$C_c = 1.15\,(e_o - 0.35) \tag{4.41}$$

Typical values of compression and swell indices as well as the corresponding void ratio at stress σ_o = 10 [kN/m²] are presented in the following table according to *Gudehus* (1981). The compression index C_c is valid for loading while C_s is valid for both heaving and reloading.

Recompression index is calculated from the plasticity index using the following correlations (*Kullhawy* and *Mayne* (1990)):

$$C_r = \frac{PI}{370} \tag{4.42}$$

where PI is the plasticity index in percent.

F.H. Kullhawy, P.H. Mayne: Manual on Estimating Soil Properties for Foundation Design, Electric Power Research Institute, EPRI, 1990.

Table 4.4 Compression and swell indices depending on the initial void ratio

Soil type	Compression index C_c [-]	Swell index C_s [-]	Initial void ratio e_o [-]
Gravely sand	0.001	0.0001	0.3
Fine sand, dense	0.005	0.0005	0.5
Fine sand, loose	0.01	0.001	0.7
Coarse silt	0.02	0.002	0.8
Clayey silt	0.03 - 0.6	0.01 - 0.02	0.9 - 1.2
Kaolin-Silt	0.1	0.03	1.5
Silt	0.1 - 0.3	0.03 - 0.1	1.2 - 2.5
Clay	0.5	0.4	5
Peat	1	0.3	10

4.9 Defining the project data

4.9.1 Firm Header

When printing the results, the main data (firm name) are displayed on each page at the top in two lines or in graphic presentation at the identification box. Firm name can be defined, modified and saved using the "Firm Header" Option from the setting Tab (see Figure 4.16).

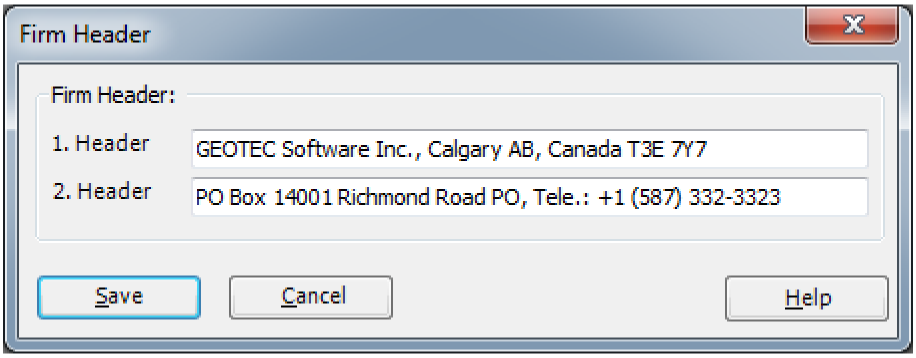


Figure 4.16 Firm Header

4.9.2 Task of the program *GEO Tools* (Analysis Type)

The program *GEO Tools* can be used to analyze various problems in Geotechnical Engineering for shallow foundations and deep foundations, Figure 4.17.

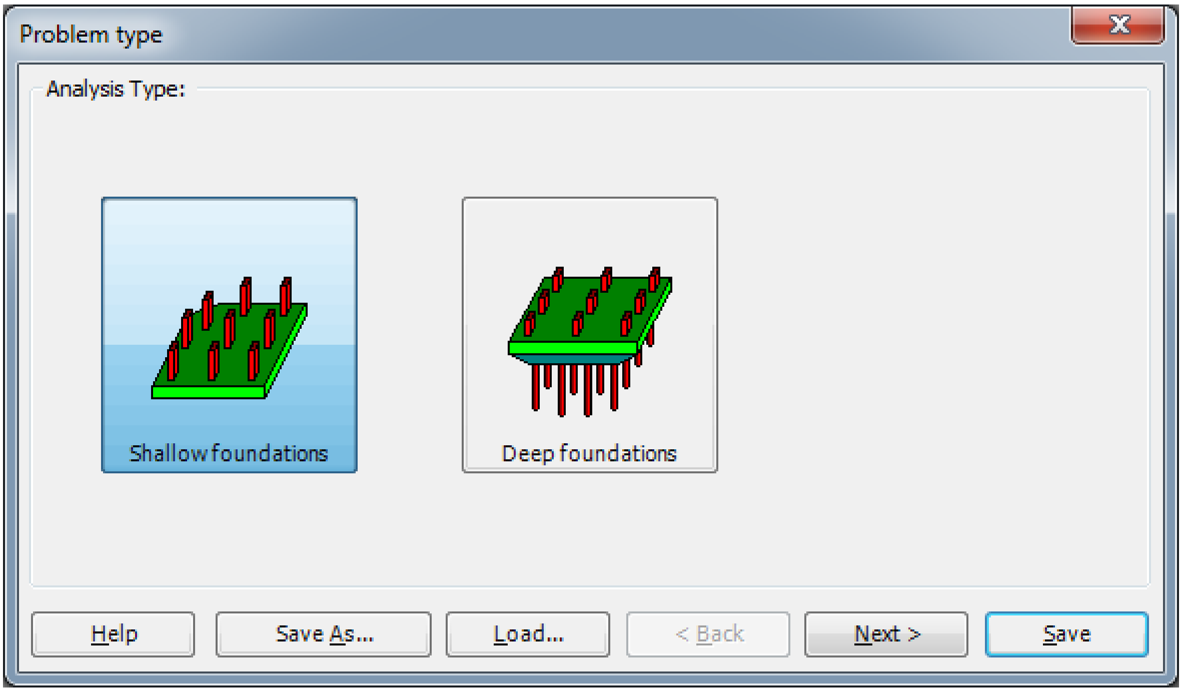


Figure 4.17 Problem type

According to the main menu in Figure 4.18 the following geotechnical problems can be calculated for shallow foundations:

1. Stresses in soil
2. Strains in soil
3. Displacements in soil
4. Consolidation settlement
5. Degree of consolidation
6. Time-settlement curve
7. Displacements of rigid raft
8. Consolidation of rigid raft
9. Settlements of footing groups

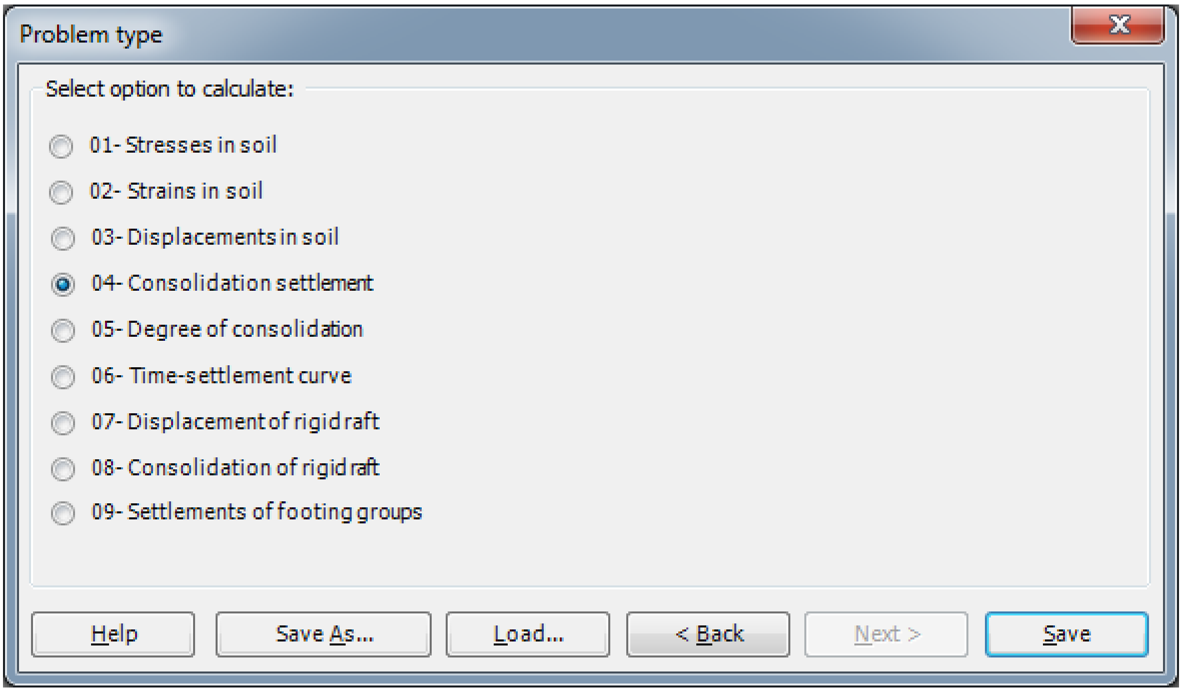


Figure 4.18 Problem type for shallow foundation

In menu of Figure 4.18 select the option:

04- Consolidation settlement

The following paragraph describes how to determine the consolidation settlement of a loaded area by the program *GEO Tools*. The input data are the geometry of the loaded area, load intensity and the properties of the soil layers.

4.9.3 Project Identification

In the program, it must be distinguished between the following two data groups:

1. System data (For identification of the project that is created and information to the output for the printer).

2. Soil data (Soil properties and so on).

The defining input data for these data groups is carried out as follows:

After clicking on the "Project Identification" Option, the following general project data are defined (Figure 4.19):

Title:	Title label
Date:	Date
Project:	Project label

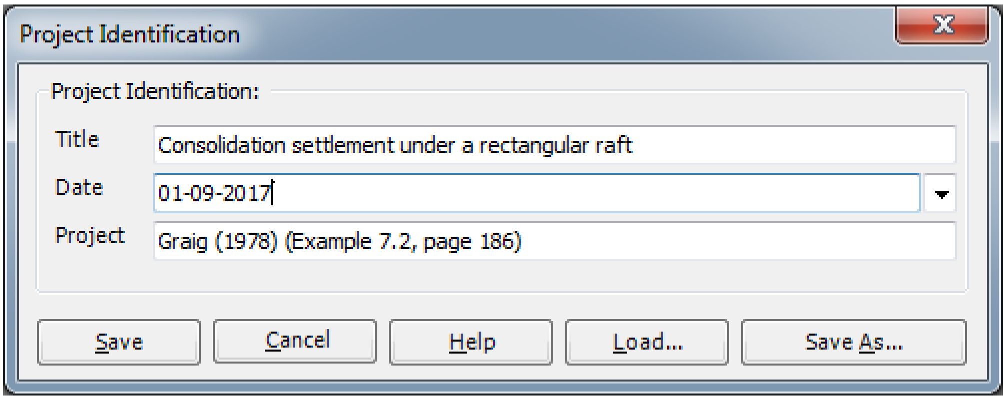


Figure 4.19 Project Identification

4.9.4 System data of the loaded area

After clicking on the "Consolidation settlement" Option, the following system data of the loaded area are defined (Figure 4.20):

Type of load:

- Point load
- Rectangular load
- Circular load
- Polygon load

Load intensity and geometry:

q	Distributed load [kN/m^2]
a	Length [m]
b	Width [m]

Coordinates of point A(x, y), under which the consolidation settlement is required to calculate and the thickness *z* of undesired layers to be neglected in the calculation:

X	X-Coordinate [m]
Y	Y-Coordinate [m]
Z	Z-Coordinate [m]

Dimensions of the element mesh:

Dx	Element length in x-direction [m]
Dy	Element length in y-direction [m]

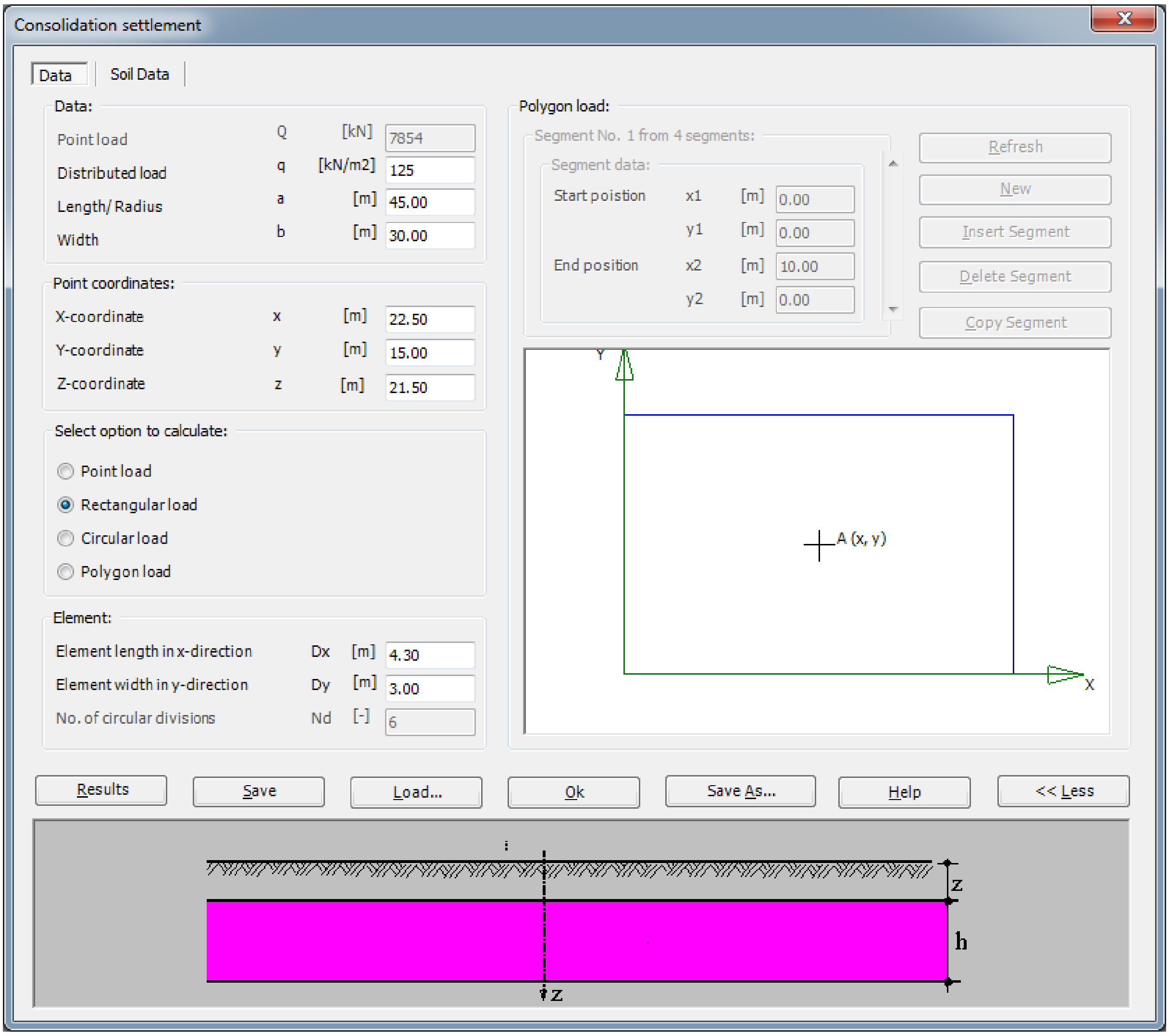


Figure 4.20 Data of the loaded area

4.9.5 Data of soil layers

Soil properties of a layer maybe defined by either Modulus of Compressibility E_s ($1/m_v$) or Compression Index (Figure 4.21):

In case of defining the soil properties by only the Modulus of compressibility E_s, unit weight of the soil and overburden pressure are not required to define. Only the layer thickness is defined.

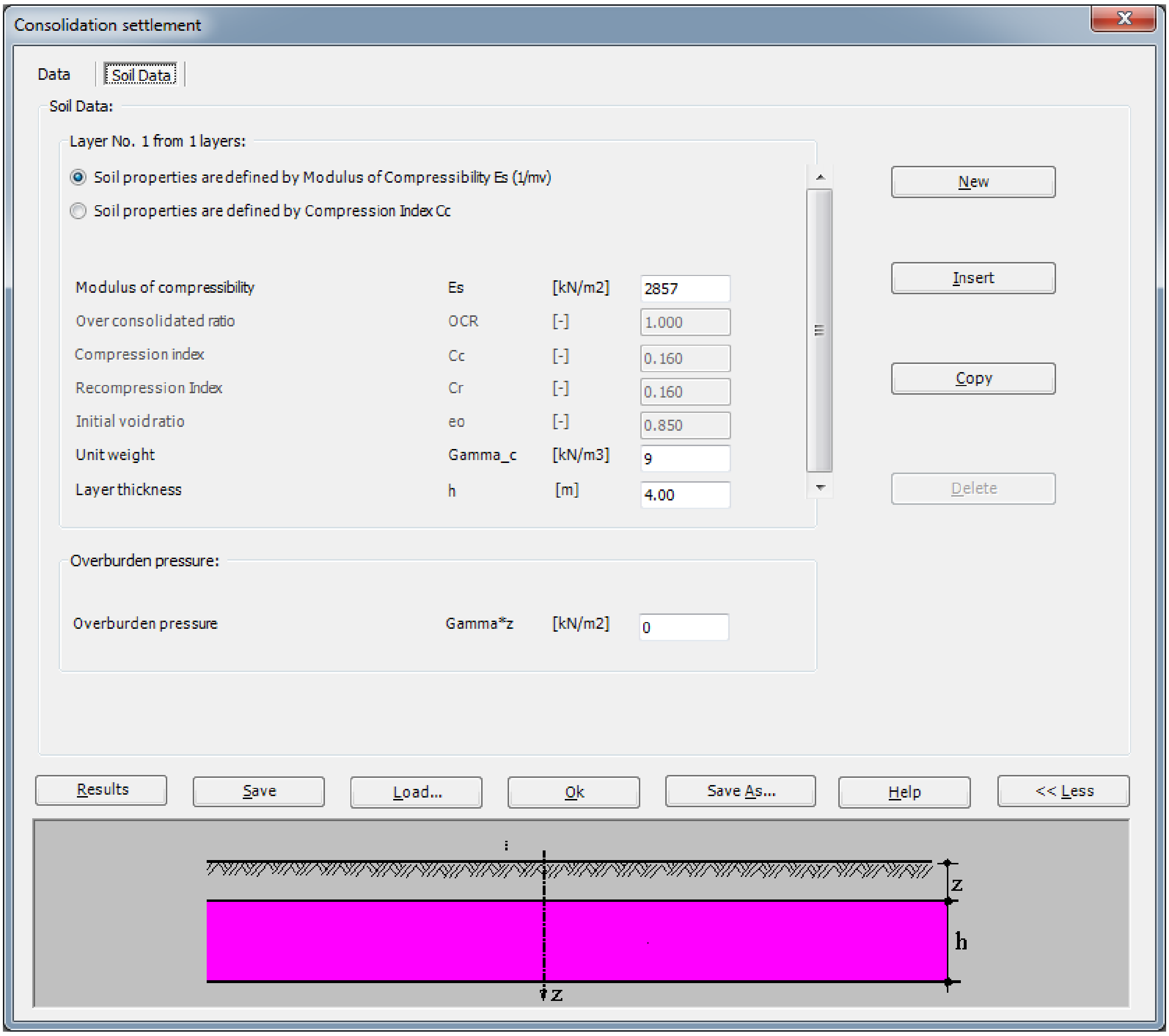


Figure 4.21 Soil data

4.10 Examples to verify consolidation settlement

4.10.1 Introduction

The application possibilities of the program *GEO Tools* to evaluate the consolidation settlement are presented below in some numerical examples. The examples were carried out to verify and test the application of the proposed numerical procedures outlined in this book.

The examples outlined in this section can be also analyzed by the program *ELPLA* and the same results can be obtained. *GEO Tools* is a simple user interface program and needs little information to define a problem. It is prefer to use it for a simple foundation geometry. Furthermore, *ELPLA* can also read data files of a consolidation problem defined by *GEO Tools*.

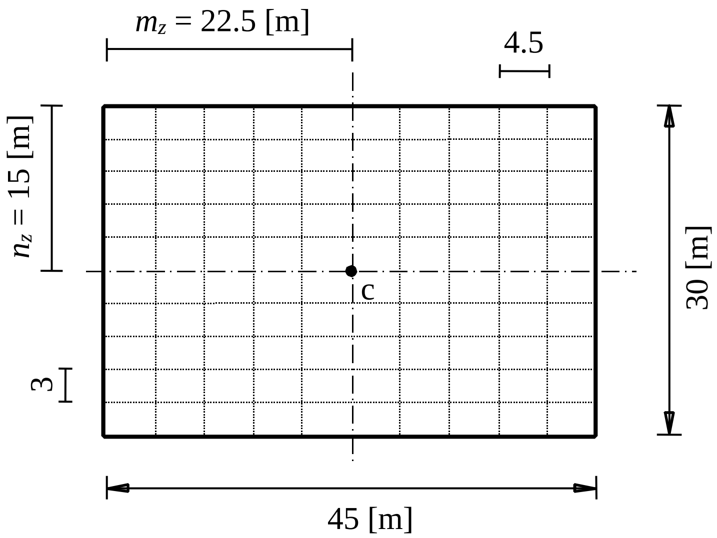
The two first examples 1 and 2 can be read and analyzed by *ELPLA* without modifications and the same results can be obtained. The other two examples 3 and 4, those having OCR >1, need to be modified after reading their data files to get the same results.

4.10.2 Example 1: Consolidation settlement under a rectangular raft

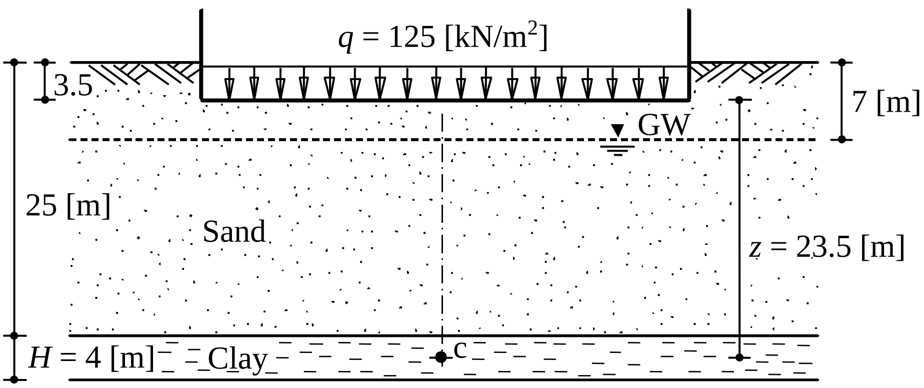
4.10.2.1 Description of the problem

To verify the consolidation settlement calculated by *GEO Tools*, the final consolidation settlement of a clay layer under a rectangular raft calculated by *Graig* (1978), Example 7.2, page 186, is compared with that obtained by *GEO Tools*.

A building supported on a raft 45 [m] × 30 [m] is considered. The contact pressure is assumed to be uniformly distributed and equal to q = 125 [kN/m^2]. The soil profile is shown in Figure 4.22. The coefficient of volume change for the clay is m_v = 0.35 [m^2/MN] (E_s = 1/ m_v = 2857 [MN/ m^2]). It is required to determine the final settlement under the center of the raft due to consolidation of the clay.



a)



b)

Figure 4.22 a) Plan of the raft with dimensions and FE-Net

b) Cross section through the soil under the raft

4.10.2.2 Hand calculation of consolidation

According to *Graig* (1978), the consolidation of the clay layer can be obtained by the hand calculation as follows:

The clay layer is thin relative to the dimensions of the raft. Therefore, it can be assumed that the consolidation is approximately one-dimensional. In this case, it will be sufficiently accurate to consider the clay layer as a whole. The consolidation settlement is to be calculated in terms of m_v. Therefore, only the effective stress increment at mid-depth of the layer is required. The increment is assumed constant over the depth of the layer. Also, $\Delta\sigma' = \Delta\sigma$ for one-dimensional consolidation and can be evaluated from *Fadum's* charts (1948), Figure 4.23.

The effective stress increment $\Delta\sigma'$ at mid-depth z = 23.5 [m] of the layer below the center of the raft is obtained as follows

$$m = \frac{m_z}{z} = \frac{22.5}{23.5} = 0.96\,[\text{-}]$$

$$n = \frac{n_z}{z} = \frac{15}{23.5} = 0.64\,[\text{-}]$$

From *Fadum's* charts (1948)

$$I_r = 0.14\ [\text{-}]$$

The effective stress $\Delta\sigma'$ is given by

$$\Delta\sigma' = 4\ I_r\ q = 4\times0.14\times125 = 70\ [\text{kN/m}^2]$$

The final consolidation settlement s_c is given by

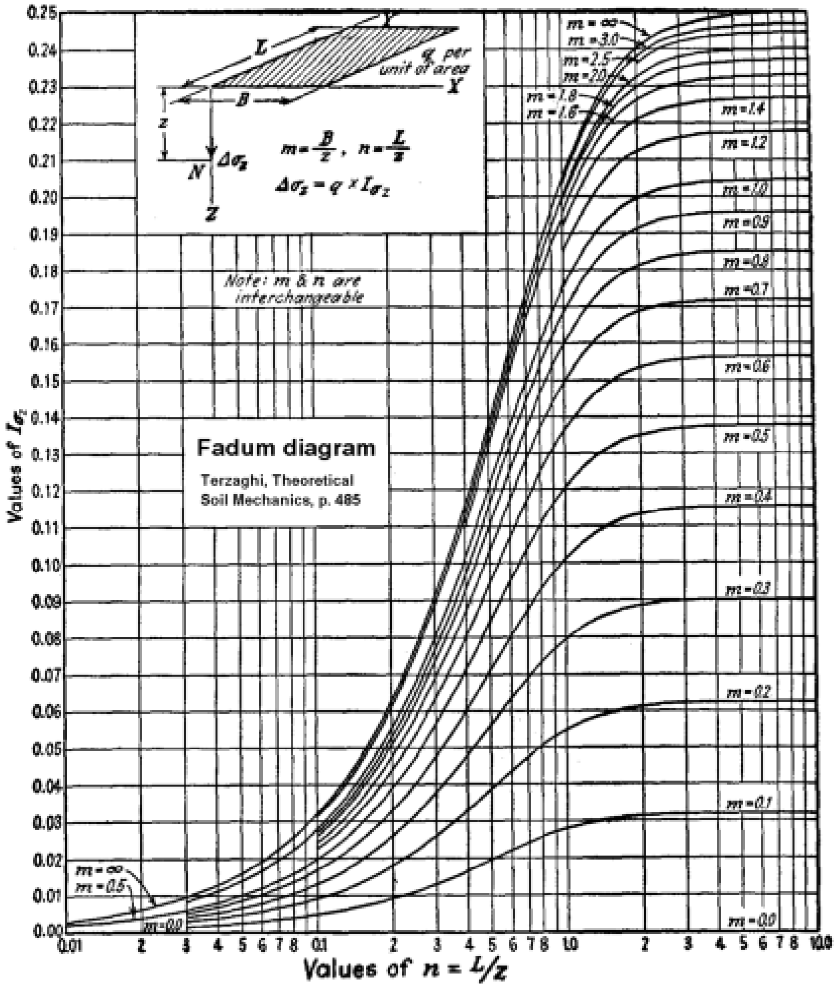
$$s_c = \Delta\sigma'\ m_v\ H = 0.35\times70\times4 = 98\ [\text{mm}] = 9.8\ [\text{cm}]$$

4.10.2.3 Consolidation by GEO Tools

The modulus of compressibility of the clay E_{s2} is obtained from the modulus of volume change m_v as

$$E_{s2} = \frac{1}{m_v} = \frac{1}{0.35} = 2.857\,[\text{MN/m}^2] = 2857\,[\text{kN/m}^2]$$

A coarse FE-Net may be chosen, where more details about the results are not required, only the settlement under the center of the raft due to consolidation of the clay. A net of equal elements is chosen. Each element has dimensions of 3 [m] × 4.5 [m] as shown in Figure 4.22a. The final consolidation settlement of the clay under the center of the raft obtained by the program *GEO Tools* is s_c = 9.8 [cm] and quite equal to that obtained by the hand calculation.



Graph for determining influence value for vertical normal stress $\Delta\sigma_z$ at point N located beneath one corner of a uniformly loaded rectangular area.

Figure 4.23 *Fadum* diagram after *Terzaghi* (1970)

The input data and results of *GEO Tools* are presented on the next pages. By comparison, one can see a good agreement with the hand calculation.

```
************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Consolidation settlement under a rectangular raft
Date: 01-09-2017
Project: Graig (1978) (Example 7.2, page 186(
File: 4Con_Ex1

------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
------------------------------------------------------------

Data :
Rectangular load                  q       [kN/m2] = 125.0
Length                            a       [m]     = 45.00
Width                             b       [m]     = 30.00

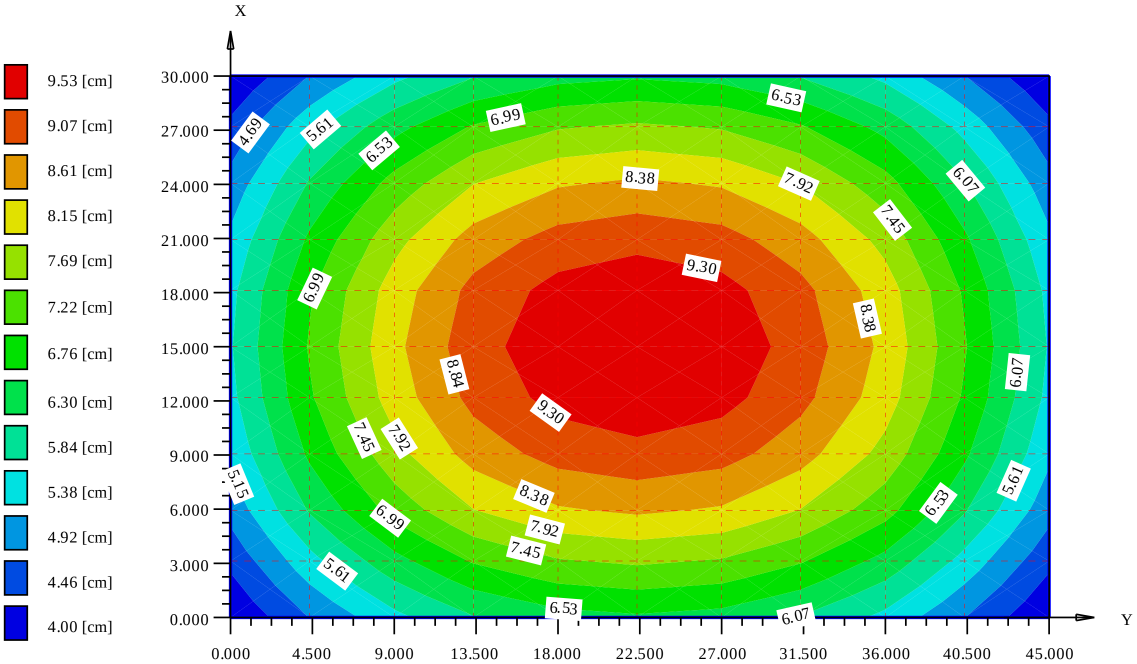
Soil Data :
Layer No.: 1
Modulus of compressibility        Es      [kN/m2] = 2857
Unit weight                       Gamma_c [kN/m3] = 9
Layer thickness                   h       [m]     = 4.00


Point coordinates :
X-coord.                          x       [m]     = 22.50
Y-coord.                          y       [m]     = 15.00
Y-coord.                          z       [m]     = 21.50

Results :
Settlement
Layer No.: 1                      S1      [cm]    = 9.76
Total                             St      [cm]    = 9.76


Settlement/ Contact pressure/ Modulus of subgrade reaction:
----------------------------------------------------------
No.   Coord.   Coord.   Settlement    Contact   Modulus of
                                     pressure     subgrade
                                                  reaction
  I        x        y            s          q           ks
[-]      [m]      [m]         [cm]    [kN/m2]      [kN/m3]
----------------------------------------------------------
 61    22.50    15.00         9.76        125         1281
----------------------------------------------------------
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 9.76 at node 61, Min. s = 3.76 at node 1

GEOTEC Software Inc., Calgary AB, Canada T3E 7Y7 PO Box 14001 Richmond Road PO, Tele.: +1 (587) 332-3323	
Scale: 311 **File: 4Con_Ex1** **Page No.:**	**Project: Graig (1978) (Example 7.2, page 186)** **Date: 01-09-2017** **Title: Consolidation settlement under a rectangular raft**

4.10.3 Example 2: Consolidation settlement under a circular footing

4.10.3.1 Description of the problem

To verify the consolidation settlement calculated by *GEO Tools*, the final consolidation settlement of a clay layer under a circular footing calculated by *Das* (1983), Example 6.3, page 371, is compared with that obtained by *GEO Tools*.

A circular footing 2 [m] in diameter at a depth of 1.0 [m] below the ground surface is considered as shown in Figure 4.24. Water table is located at 1.5 [m] below the ground surface. The contact pressure under the footing is assumed to be uniformly distributed and equal to q = 150 [kN/m^2]. A normally consolidated clay layer 5 [m] thick is located at a depth of 2.0 [m] below the ground surface. The soil profile is shown in Figure 4.24, while the soil properties are shown in Table 4.5. It is required to determine the final settlement under the center of the footing due to consolidation of the clay.

Table 4.5 Soil properties

Layer No.	Type of Soil	Depth of the layer under the ground surface z [m]	Unit weight of the soil γ [kN/m^3]	Compression index C_c [-]	Void ratio e_o [-]
1	Sand	1.5	17.00	-	-
2	Sand	2.0	9.19	-	-
3	Clay	7.0	8.69	0.16	0.85

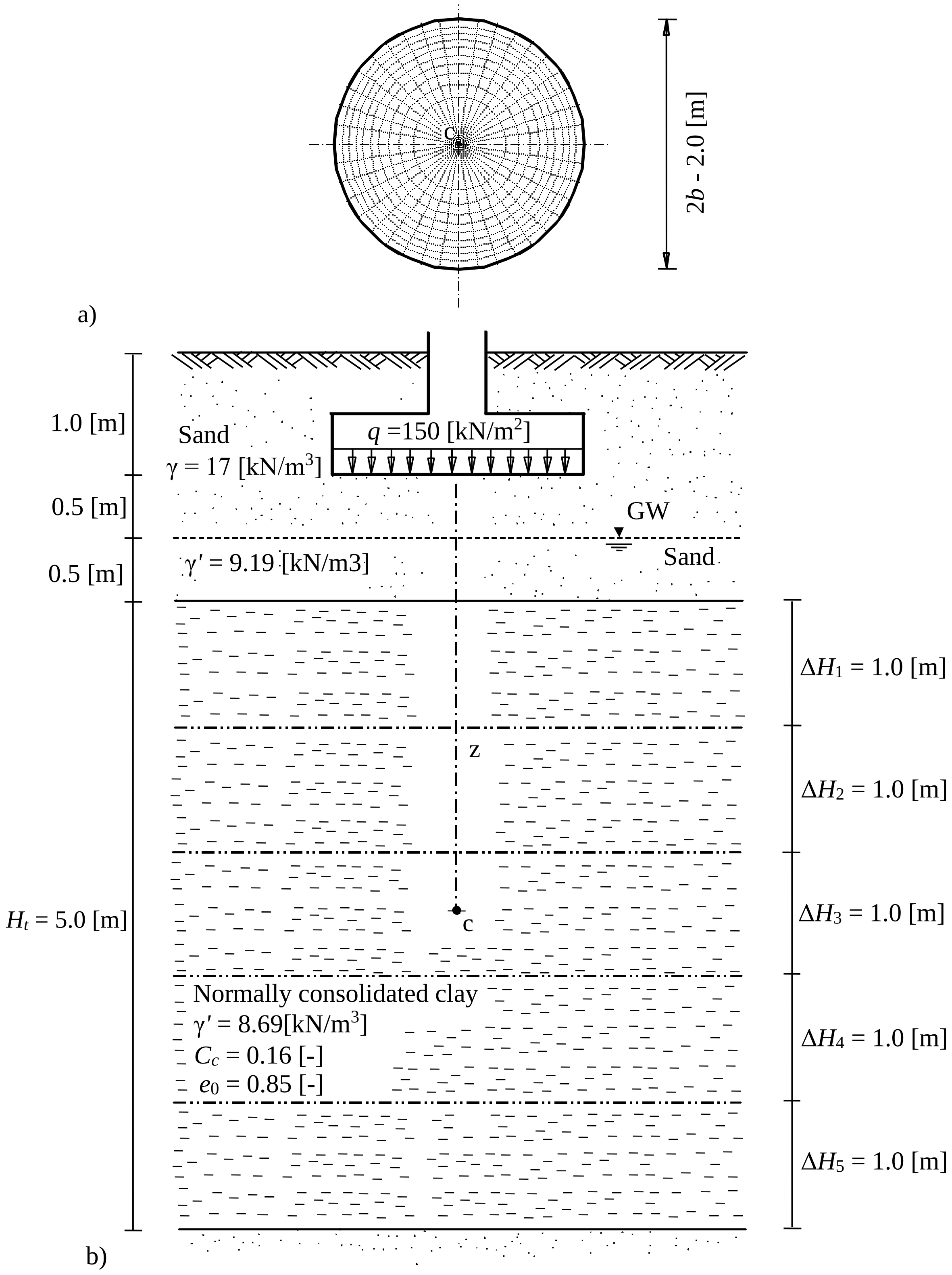


Figure 4.24 a) Plan of the footing with dimensions and FE-Net

b) Cross section through the soil under the footing

4.10.3.2 Hand calculation of consolidation

According to *Das* (1983), the consolidation of the clay layer can be obtained by the hand calculation as follows:

The clay layer is thick relative to the dimensions of the footing. Therefore, the clay layer is divided into five layers each 1.0 [m] thick.

4.10.3.2.1 Calculation of the effective stress $\sigma'_{o(i)}$

The effective stress σ'_o on the top of the clay layer is

$$\sigma'_o = \gamma_1 \, z_1 + \gamma_2 \, z_2$$

$$\sigma'_o = 17 \times 1.5 + 9.19 \times 0.5 = 30.1 \, [\mathrm{kN/m^2}]$$

The effective stress $\sigma'_{o(1)}$ at the middle of the first layer is

$$\sigma'_{o(1)} = \gamma_1 \, z_1 + \gamma_2 \, z_2 + \gamma_3 \, \frac{\Delta H_1}{2}$$

$$\sigma'_{o(1)} = 17 \times 1.5 + 9.19 \times 0.5 + 8.69 \times \frac{1}{2} = 34.44 \, [\mathrm{kN/m^2}]$$

The effective stress $\sigma'_{o(2)}$ at the middle of the second layer is

$$\sigma'_{o(2)} = \sigma'_{o(1)} + \gamma_3 \left(\frac{\Delta H_1}{2} + \frac{\Delta H_2}{2} \right)$$

$$\sigma'_{o(2)} = 34.44 + 8.69 \left(\frac{1}{2} + \frac{1}{2} \right) = 43.13 \, [\mathrm{kN/m^2}]$$

Similarly

$$\sigma'_{o(3)} = 43.13 + 8.69 = 51.82 \, [\mathrm{kN/m^2}]$$

$$\sigma'_{o(4)} = 51.82 + 8.69 = 60.51 \, [\mathrm{kN/m^2}]$$

$$\sigma'_{o(5)} = 60.51 + 8.69 = 69.20 \, [\mathrm{kN/m^2}]$$

4.10.3.2.2 Calculation of the increase of effective stress $\Delta\sigma'_i$

For a circular loaded area of radius *b* and load *q*, the increase of effective stress $\Delta\sigma'_i$ below the center at depth *z* is given by (*Das* (1983))

$$\Delta\sigma'_i = q\left(1 - \frac{1}{\left[\left(\frac{b}{z}\right)^2 + 1\right]^{3/2}}\right)$$

Hence

$$\Delta\sigma'_1 = 150\left(1 - \frac{1}{\left[\left(\frac{1}{1.5}\right)^2 + 1\right]^{3/2}}\right) = 63.59\,[\mathrm{kN/m^2}]$$

$$\Delta\sigma'_2 = 150\left(1 - \frac{1}{\left[\left(\frac{1}{2.5}\right)^2 + 1\right]^{3/2}}\right) = 29.93\,[\mathrm{kN/m^2}]$$

$$\Delta\sigma'_3 = 150\left(1 - \frac{1}{\left[\left(\frac{1}{3.5}\right)^2 + 1\right]^{3/2}}\right) = 16.66\,[\mathrm{kN/m^2}]$$

$$\Delta\sigma'_4 = 150\left(1 - \frac{1}{\left[\left(\frac{1}{4.5}\right)^2 + 1\right]^{3/2}}\right) = 10.46\,[\mathrm{kN/m^2}]$$

$$\Delta\sigma'_5 = 150\left(1 - \frac{1}{\left[\left(\frac{1}{5.5}\right)^2 + 1\right]^{3/2}}\right) = 7.14\,[\mathrm{kN/m^2}]$$

4.10.3.2.3 Calculation of consolidation settlement s_c

The steps of the calculation of consolidation settlement s_c are given in Table 4.6 and Figure 4.24.

Table 4.6 Steps of calculation of consolidation settlement s_c

Layer No.	Layer thickness ΔH_i [m]	Effective stress $\sigma'_{o(i)}$ [kN/m^2]	Increase of effective stress $\Delta\sigma'_i$ [kN/m^2]	Decrease of void ratio $\Delta e_{(i)}$ [-]	Consolidation settlement $s_{c(i)}$ [m]
1	1.0	34.44	63.59	0.07270	0.0393
2	1.0	43.13	29.93	0.03660	0.0198
3	1.0	51.82	16.66	0.01940	0.0105
4	1.0	60.51	10.46	0.01110	0.0060
5	1.0	69.20	7.14	0.00682	0.0037
Total consolidation settlement Σ					0.0793

In Table 4.6 the decrease of void ratio $\Delta e_{(i)}$ and the consolidation settlement $s_{c(i)}$ are given by

$$\Delta e_{(i)} = c_c \log\left(\frac{\sigma'_{o(i)} + \Delta\sigma'_{o(i)}}{\sigma'_{o(i)}}\right)$$

$$s_{c(i)} = \frac{\Delta e_{(i)}}{1+e_o}\,\Delta H_{(i)}$$

The total consolidation settlement obtained by the hand calculation is

$$s_c = 0.0793 \text{ [m]} = 7.93 \text{ [cm]}$$

4.10.3.3 Consolidation by GEO Tools

GEO Tools can consider the clay layer as a whole and calculate the consolidation settlement directly in terms of Compression index C_c and Void ratio e_o. The final consolidation settlement of the clay under the center of the footing obtained by the program *GEO Tools* is s_c = 8.01 [cm] and nearly equal to that obtained by the hand calculation. The input data and results of *GEO Tools* are presented on the next pages.

```
************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Consolidation settlement under a circular footing
Date: 02.01.2005
Project: Das (1983) (Example 6.3, page 371(
File: 4Con_Ex2

------------------------------------------------------------
Consolidation settlement in soil due to circular load
------------------------------------------------------------

Data :
Circular load                         p        [kN/m2] = 150.0
Radius                                a        [m]     = 1.00

Soil Data :
Layer No.: 1
Compression index                     Cc       [-]     = 0.160
Recompression Index                   Cr       [-]     = 0.160
Over consolidated ratio               OCR      [-]     = 1.000
Initial void ratio                    eo       [-]     = 0.850
Unit weight                           Gamma_c  [kN/m3] = 9
Layer thickness                       h        [m]     = 5.00

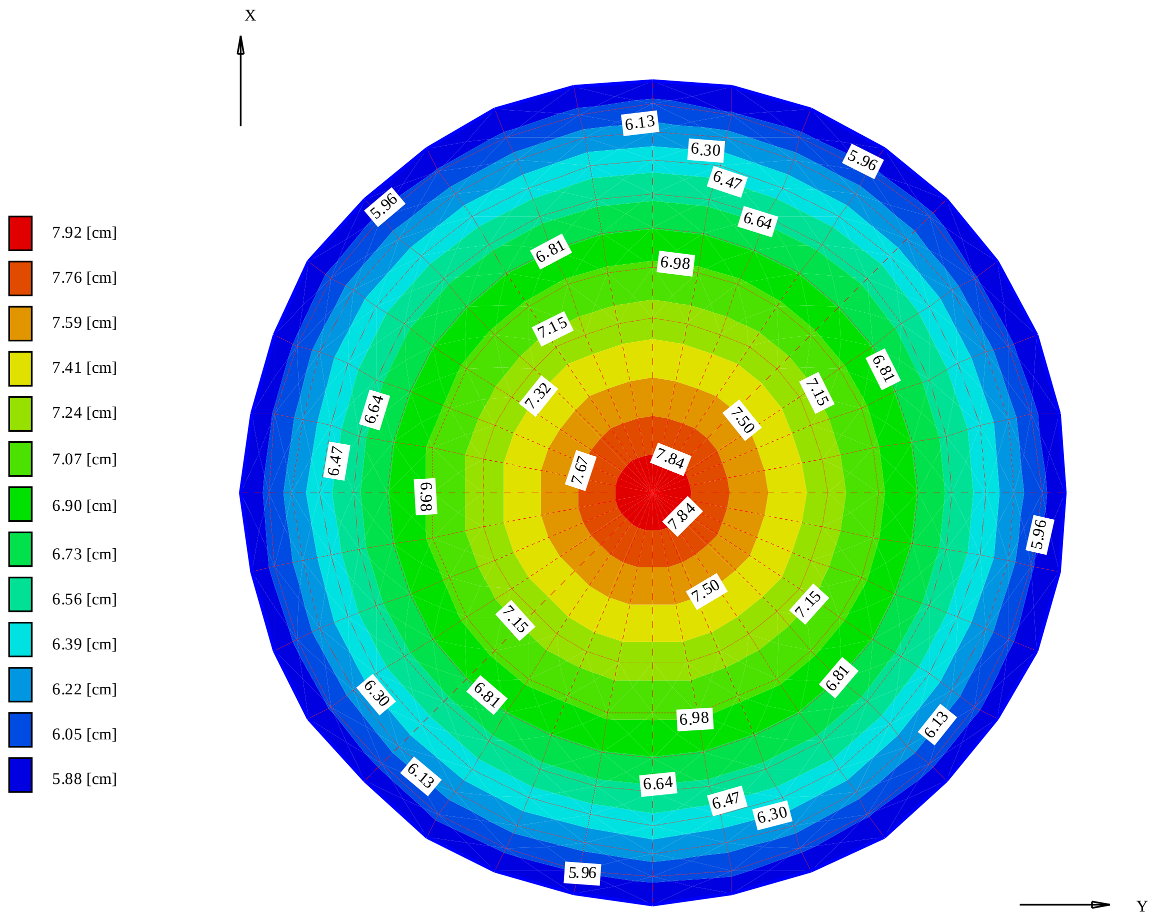

Overburden pressure                   Gamma*z  [kN/m2] = 30.10
Depth increment in z-direction        Dz       [m]     = 0.00

Point coordinates :
X-coord.                              x        [m]     = 1.00
Y-coord.                              y        [m]     = 1.00
Y-coord.                              z        [m]     = 1.00

Results :
Settlement
Layer No.: 1                          S1       [cm]    = 8.01
Total                                 St       [cm]    = 8.01


Settlement/ Contact pressure/ Modulus of subgrade reaction:
----------------------------------------------------------
No.   Coord.   Coord.   Settlement    Contact   Modulus of
                                     pressure     subgrade
                                                  reaction
  I        x        y            s          q           ks
[-]      [m]      [m]         [cm]    [kN/m2]      [kN/m3]
----------------------------------------------------------
  1     1.00     1.00         8.02        150         1870
----------------------------------------------------------
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 8.01 at node 1, Min. s = 5.79 at node 41

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Scale: 13 **File: 4Con_Ex2** **Page No.:**	**Project: Das (1983) (Example 6.3, page 371)** **Date: 02.01.2005** **Title: Consolidation settlement under a circular footing**

4.10.4 Example 3: Loading and reloading settlement under a square raft

4.10.4.1 Description of the problem

To verify the consolidation settlement calculated by the program *GEO Tools* in the cases of loading and reloading pressures, a hand calculation of a consolidation settlement for a clay layer under a square raft is compared with that obtained by *GEO Tools*.

A square raft of B = 20 [m] side located at a depth of d_f =5 [m] below the ground surface is considered as shown in Figure 4.25. Ground water table lies at a depth z = 5 [m] under the ground surface. The contact pressure under the raft is assumed to be uniformly distributed and equal to q = 200 [kN/m^2]. A clay layer of H = 10 [m] thick is located directly under the raft. Figure 4.25 shows raft with dimensions and soil profile, while the soil data are shown Table 4.7. It is required to determine the final settlement under the center of the raft due to consolidation of the clay taking into account the reloading pressure due to the excavation weight.

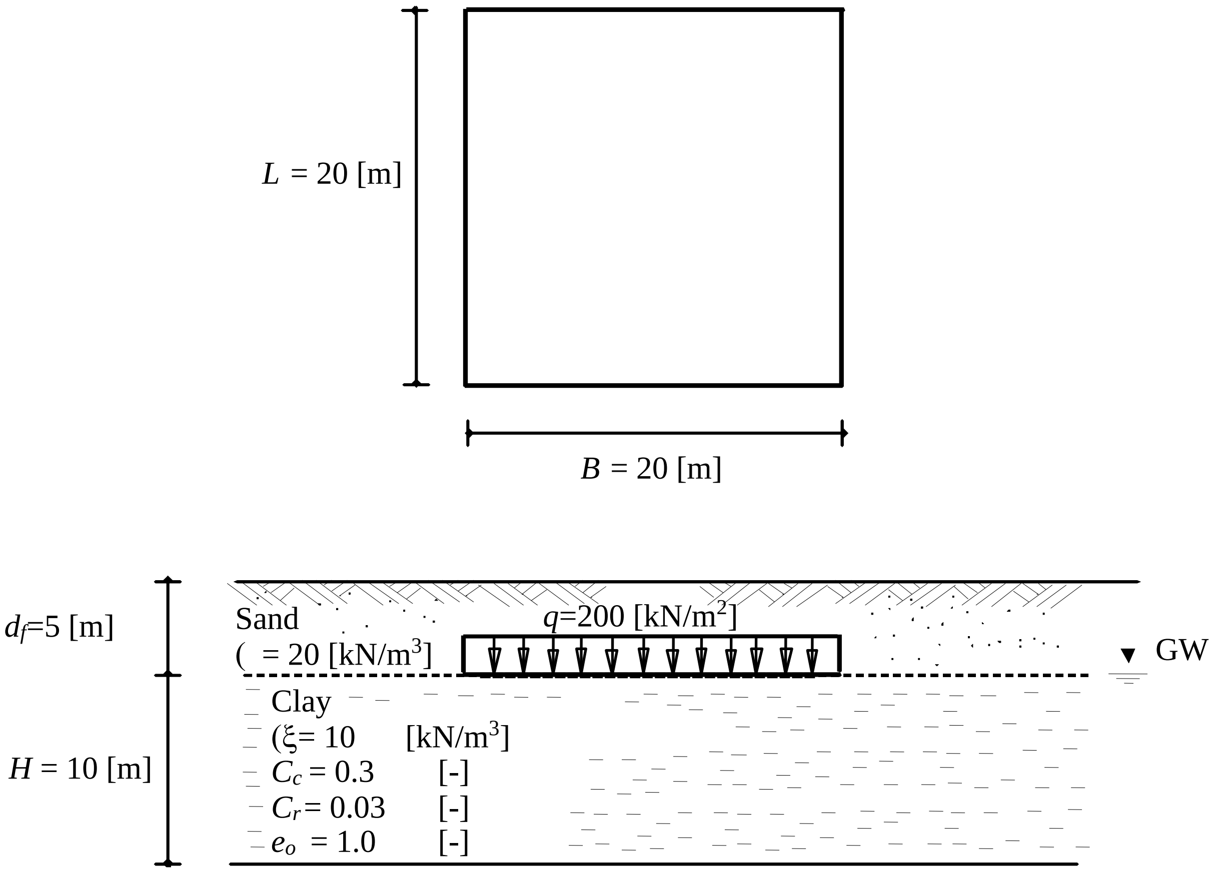


Figure 4.25 Raft with dimensions and soil profile

Table 4.7 Soil data of the clay layer

Unit weight of the soil above the water level	γ	= 20	[kN/m^3]
Unit weight of the soil under the water level	$\gamma\xi$	= 10	[kN/m^3]
Recompression index	C_r	= 0.03	[-]
Compression index	C_c	= 0.3	[-]
Initial void ratio	e_o	= 1.0	[-]

4.10.4.2 Hand calculation

4.10.4.2.1 Loading and reloading pressure

The reloading pressure due to the soil excavation is $q_v = \gamma.\, d_f = 20 \times 5 = 100$ [kN/m^2].
The total applied pressure on the soil due to the raft is $q = 200$ [kN/m^2].

4.10.4.2.2 Overburden pressure

The overburden pressure on the top of the clay layer is given by:

$$\sigma_o = \gamma d_f = 20 \times 5 = 100\,[\text{kN/m}^2]$$

The overburden pressure in the middle of the clay layer is given by:

$$\sigma_o = \gamma d_f + \frac{\gamma' H}{2} = 20 \times 5 + \frac{10 \times 10}{2} = 150\,[\text{kN/m}^2]$$

4.10.4.2.3 Increment of vertical stress

For a rectangular loaded area of intensity q [kN/m^2] with sides a [m] and b [m] acting on the surface, the increment of vertical stress $\Delta\sigma$ [kN/m^2] in the soil layer of thickness h under the corner of the rectangular can be expressed as:

$$\Delta\sigma = \frac{q}{2\pi h}\left\{\left(b \ln\frac{(c_2 - a)(m + a)}{(c_2 + a)(m - a)} + a \ln\frac{(c_2 - b)(m + b)}{(c_2 + b)(m - b)} + h_2 \tan^{-1}\frac{a\,b}{h_2\,c_2}\right) - \left(b \ln\frac{(c_1 - a)(m + a)}{(c_1 + a)(m - a)} + a \ln\frac{(c_1 - b)(m + b)}{(c_1 + b)(m - b)} + h_1 \tan^{-1}\frac{a\,b}{h_1\,c_1}\right)\right\}$$

where: $m = \sqrt{a^2 + b^2}$, $c_2 = \sqrt{a^2 + b^2 + h_2^2}$ and $c_1 = \sqrt{a^2 + b^2 + h_1^2}$

h Thickness of the soil layer, [m].
h_1 Depth of the layer top from the surface, [m].
h_2 Depth of the layer bottom from the surface, [m].

when $h_1 = 0$, the above equation becomes:

$$\Delta\sigma = \frac{q}{2\pi h}\left\{\left(2b \ln\frac{(c_2 - a)(m + a)}{(c_2 + a)(m - a)} + h_2 \tan^{-1}\frac{a\,b}{h_2\,c_2}\right)\right\}$$

The stress under the corner of the quarter of the raft is:

$$\Delta\sigma = \frac{q}{2\pi 10}\left\{\left(2\times 10\ln\frac{(\sqrt{300}-10)(\sqrt{200}+10)}{(\sqrt{300}+10)(\sqrt{200}-10)}+10\tan^{-1}\frac{10\times 10}{10\times\sqrt{300}}\right)\right\} = I_\sigma q$$

$$\Delta\sigma = 0.225q$$

$\therefore$ The increment of vertical stress in the soil layer due to the reloading pressure q_v is given by:

$$\Delta\sigma_{\mathrm{av1}} = 4I_\sigma q_v = 4\times 0.225\times 100 = 90\ [\mathrm{kN/m^2}]$$

$\therefore$ Increment of vertical stress in the soil layer due to the total applied pressure q is given by:

$$\Delta\sigma_{\mathrm{av}} = 4I_\sigma q = 4\times 0.225\times 200 = 180\ [\mathrm{kN/m^2}]$$

4.10.4.2.4 Over consolidated ratio OCR

Preconsolidation pressure of the layer σ_c is given by:

$$\sigma_c = \sigma_o + \Delta\sigma_{va1} = 150 + 90 = 240\ [\mathrm{kN/m^2}]$$

Over consolidation ratio *OCR* is given by:

$$OCR = \frac{\sigma_c}{\sigma_o} = \frac{240}{150} = 1.6$$

$OCR > 1$: The clay is over consolidated.

4.10.4.2.5 Consolidation settlement:

The total settlement of the raft occurred due to two parts. Reloading part due to the reloading pressure of q_v = 100 [kN/m^2], while loading part due to the pressure from q_v = 100 [kN/m^2] to the total pressure q = 200 [kN/m^2]. This settlement can be obtained from the following equation:

$$Sc = \frac{C_r h}{1+e_o}\log\left(\frac{\sigma_o+\Delta\sigma_{va1}}{\sigma_o}\right) + \frac{C_c h}{1+e_o}\log\left(\frac{\sigma_o+\Delta\sigma_{va}}{\sigma_o+\Delta\sigma_{va1}}\right)$$

where:

Sc	Consolidation settlement, [m].
h	Layer thickness, [m].
C_r	Recompression index, [-].
C_c	Compression index, [-].
e_o	Initial void ratio, [-].
σ_o	Overburden pressure in the middle of the layer, [kN/m^2].

$\Delta\sigma_{va}$ Increment of vertical stress in the soil layer due to the total applied pressure q, [kN/m^2].

$\Delta\sigma_{va1}$ Increment of vertical stress in the soil layer due to the reloading pressure q_v, [kN/m^2].

The consolidation settlement is given by:

$$S_c = \frac{0.03\times 10}{1+1}\log\left(\frac{150+90}{150}\right) + \frac{0.3\times 10}{1+1}\log\left(\frac{150+180}{150+90}\right)$$

$$S_c = 0.0306 + 0.2075 \text{ [m]}$$

$$S_c = 3.06 + 20.75 = 23.81 \text{ [cm]}$$

where:

Reloading settlement is	S_u = 3.06 [cm]
Loading settlement is	S_e = 20.75 [cm]
Total consolidation settlement is	S_c = 23.81 [cm]

4.10.4.3 Consolidation settlement by GEO Tools

The consolidation settlement obtained from *GEO Tools* at the center of the raft is 23.84 [cm] nearly the same as that of the hand calculation with a difference 0.03 [cm]. The input data and results of *GEO Tools* are presented on the next pages.

```
************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Consolidation settlement under a square raft
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex3

------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
------------------------------------------------------------

Data :
Rectangular load                      q       [kN/m2] = 200.0
Length                                a       [m]     = 20.00
Width                                 b       [m]     = 20.00

Soil Data :
Layer No.: 1
Compression index                     Cc      [-]     = 0.300
Recompression Index                   Cr      [-]     = 0.030
Over consolidated ratio               OCR     [-]     = 1.600
Initial void ratio                    eo      [-]     = 1.000
Unit weight                           Gamma_c [kN/m3] = 10
Layer thickness                       h       [m]     = 10.00

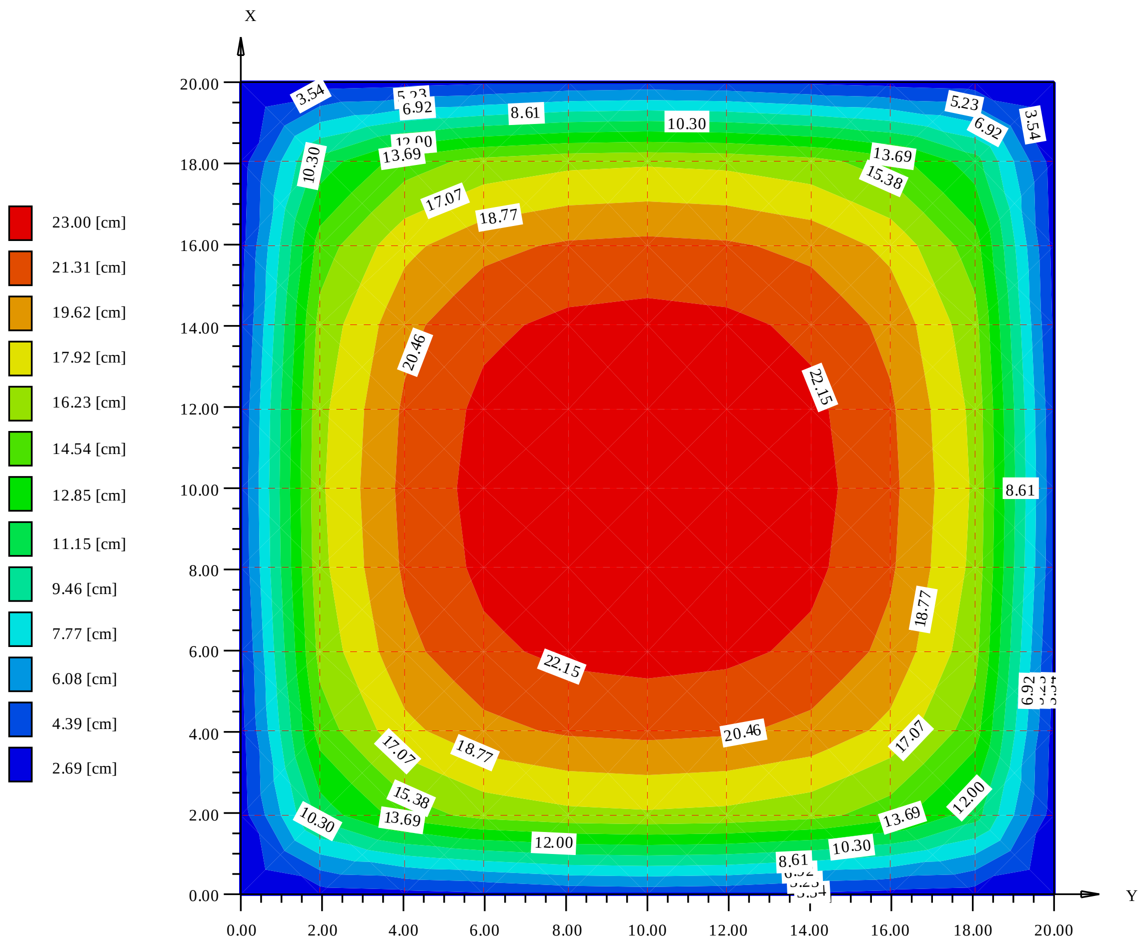

Overburden pressure                   Gamma*z [kN/m2] = 100.00
Depth increment in z-direction        Dz      [m]     = 0.00

Point coordinates :
X-coord.                              x       [m]     = 10.00
Y-coord.                              y       [m]     = 10.00
Y-coord.                              z       [m]     = 0.00

Results :
Settlement
Layer No.: 1                          S1      [cm]    = 23.84
Total                                 St      [cm]    = 23.84


Settlement/ Contact pressure/ Modulus of subgrade reaction:
----------------------------------------------------------
No.   Coord.   Coord.   Settlement    Contact   Modulus of
                                     pressure     subgrade
                                                  reaction
  I        x        y            s          q           ks
[-]      [m]      [m]         [cm]    [kN/m2]      [kN/m3]
----------------------------------------------------------
 61    10.00    10.00        23.84        200          839
----------------------------------------------------------
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 23.84 at node 61, Min. s = 1.84 at node 1

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Scale: 139 **File: 4Con_Ex3** **Page No.:**	**Project: User's Manual** **Date: 01-09-2017** **Title: Consolidation settlement under a square raft**

4.10.5 Example 4: Preconsolidated settlement under a rectangular raft

4.10.5.1 Description of the problem

To verify the rigid settlement for a preconsolidated clay layer calculated by the program *GEO Tools*, a hand calculation of a rigid settlement for two different layers under a rectangular raft is compared with that obtained by *GEO Tools*.

A tower of 10 floors supported on a raft 32.5 [m] × 23.5 [m] located at a depth of d_f =1.5 [m] below the ground surface is considered as shown in Figure 4.26. Ground water table lies directly under the raft. The contact pressure is assumed to be uniformly distributed and equal to q = 120 [kN/m^2]. The soil profile consists of a sand layer having a thickness of 10.5 [m] underlain by a clay layer of 14 [m] thickness as shown in Figure 4.26. The soil data of the two layers are listed in Table 4.8 and Table 4.9. It is required to determine the expected rigid settlement of the raft.

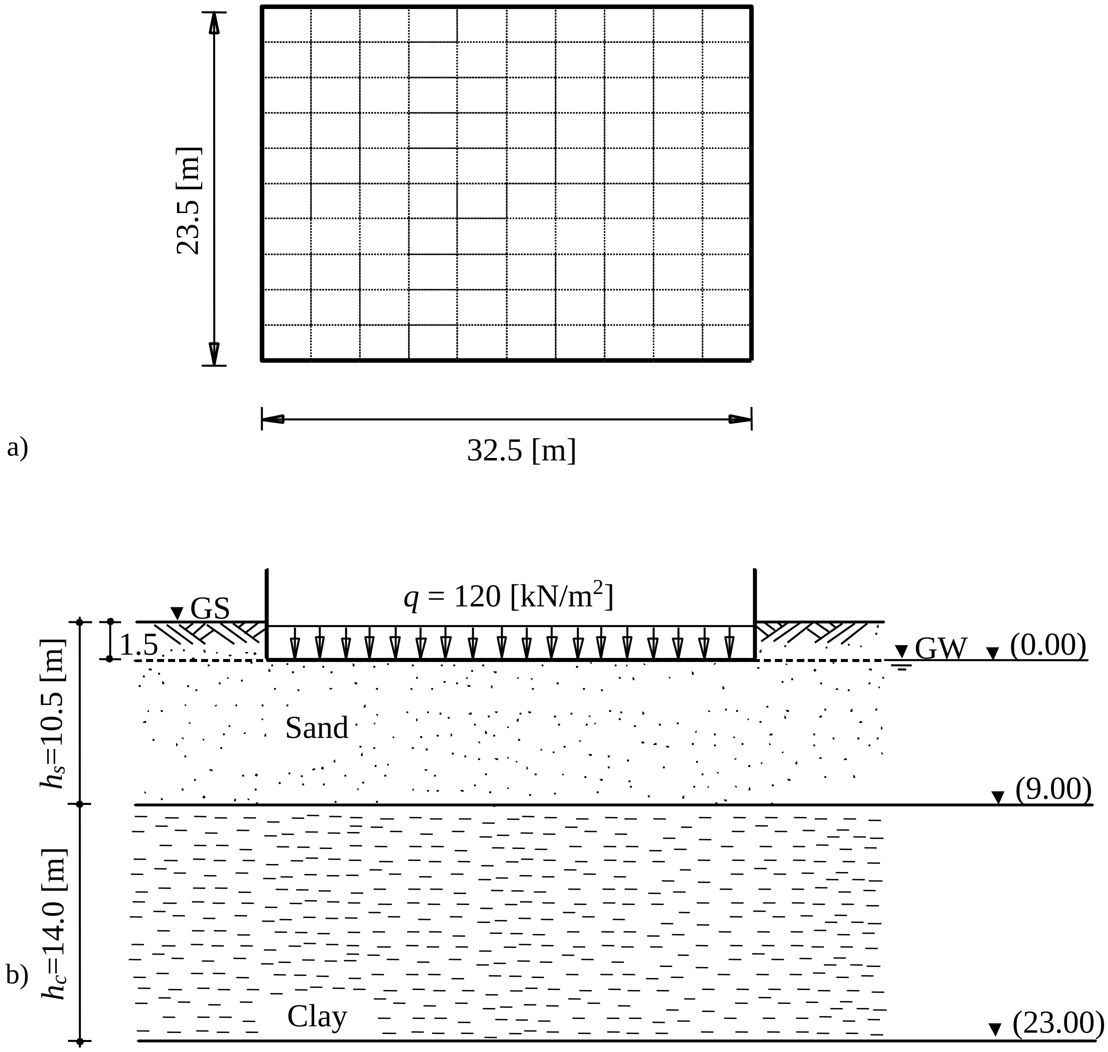


Figure 4.26 a) Plan of the raft with dimensions and FE-Net

b) Cross section through the soil under the raft

Table 4.8 Soil data of the sand layer

Unit weight of the soil above the water level	γ_s	= 16	[kN/m^3]
Unit weight of the soil under the water level	$\gamma_s\xi$	= 7	[kN/m^3]
Modulus of Compressibility	E_s	= 14000	[kN/m^2]

Table 4.9 Soil data of the clay layer

Unit weight of the soil under the water level	$\gamma_c\xi$	= 6	[kN/m^3]
Recompression index	C_r	= 0.15	[-]
Compression index	C_c	= 0.75	[-]
Initial void ratio	e_o	= 1.5	[-]

4.10.5.2 Hand calculation

The definition of the characteristic point according to *Graßhoff* (1955) can be used to determine the rigid settlement of the raft. The characteristic point o is defined as that point of a surface area loaded by a uniformly distributed pressure, where the settlement s_o due to that pressure is identical with the displacement w_o of rigid foundation of the same shape and loading. For a rectangular foundation, the characteristic point takes the coordinates a_c =0.87A and b_c = 0.87B, where A and B are the foundation dimensions.

The settlement is obtained for each layer at the characteristic point o by the hand calculation. This point o takes the coordinates a_c = 0.87 A and b_c = 0.87 B as shown in Figure 4.27. The raft is divided into four rectangular areas *I*, *II*, *III* and *IV* as shown in Figure 4.27. The settlement of point o is then the sum of settlements of areas *I*, *II*, *III* and *IV*.

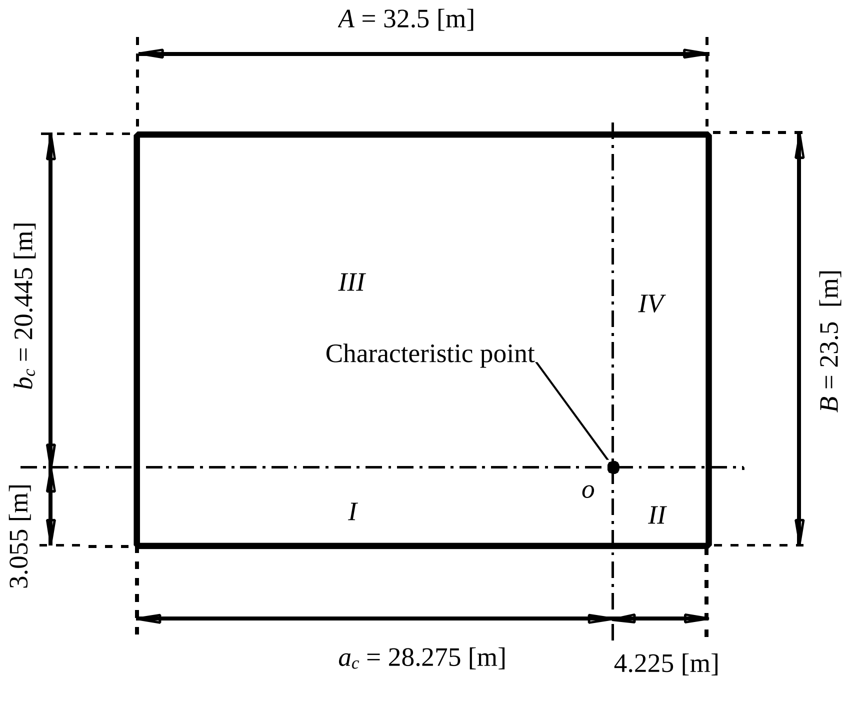


Figure 4.27 Characteristic point o of the settlement on the raft

4.10.5.2.1 Settlement of the sand layer

For a rectangular loaded area of intensity q [kN/m^2] with sides a [m] and b [m] acting on the surface, the settlement Sc under the corner for a soil layer defined by E_s can be expressed as:

$$Sc = \frac{q}{2\pi E_s}\left(b\ln\frac{(c-a)(m+a)}{(c+a)(m-a)} + a\ln\frac{(c-b)(m+b)}{(c+b)(m-b)} + h\tan^{-1}\frac{ab}{hc} \right)$$

where: $m = \sqrt{a^2+b^2}$, $c = \sqrt{a^2+b^2+h^2}$ and h is the thickness of the soil layer, [m].

The settlement due to the loaded area I under the characteristic point o of the raft is:

$$m = \sqrt{a^2+b^2} = \sqrt{28.275^2+3.055^2} = 28.440$$
$$c = \sqrt{a^2+b^2+h^2} = \sqrt{28.275^2+3.055^2+9^2} = 29.830$$

$$Sc_{sI} = \frac{120}{2\pi 14000}\left\{\left(3.055\ln\frac{(29.830-28.275)(28.440+28.275)}{(29.830+28.275)(28.440-28.275)}+\right.\right.$$
$$\left.\left.+28.275\ln\frac{(29.830-3.055)(28.440+3.055)}{(29.830+3.055)(28.440-3.055)}++9\tan^{-1}\frac{28.275\times 3.055}{9\times 29.830}\right)\right\}$$
$$Sc_{sI} = 0.0135\,[\mathrm{m}]$$

The settlement due to the loaded area II under the characteristic point o of the raft is:

$$m = \sqrt{a^2+b^2} = \sqrt{4.225^2+3.055^2} = 5.214$$
$$c = \sqrt{a^2+b^2+h^2} = \sqrt{4.225^2+3.055^2+9^2} = 10.401$$

$$Sc_{sII} = \frac{120}{2\pi 14000}\left\{\left(3.055\ln\frac{(10.401-4.225)(5.214+4.225)}{(10.401+4.255)(5.214\ -4.225)}+\right.\right.$$
$$\left.\left.+4.225\ln\frac{(10.401-3.055)(5.214+3.055)}{(10.401+3.055)(5.214\ -3.055)}++9\tan^{-1}\frac{4.225\times 3.055}{9\times 10.401}\right)\right\}$$
$$Sc_{sII} = 0.0117\,[\mathrm{m}]$$

The settlement due to the loaded area III under the characteristic point o of the raft is:

$$m = \sqrt{a^2+b^2} = \sqrt{28.275^2+20.445^2} = 34.892$$
$$c = \sqrt{a^2+b^2+h^2} = \sqrt{28.275^2+20.445^2+9^2} = 36.034$$

$$Sc_{sIII} = \frac{120}{2\pi\,14000}\left\{\left(20.445\ln\frac{(36.034-28.275)\,(34.892+28.275)}{(36.034+28.275)\,(34.892-28.275)}+\right.\right.$$
$$\left.\left.+28.275\ln\frac{(36.034-20.445)\,(34.892+20.445)}{(36.034+20.445)\,(34.892-20.445)}++9\tan^{-1}\frac{28.275\times 20.445}{9\times 36.034}\right)\right\}$$
$$Sc_{sIII} = 0.0191[\text{m}]$$

The settlement due to the loaded area *IV* under the characteristic point *o* of the raft is:

$$m=\sqrt{a^2+b^2}=\sqrt{4.225^2+20.445^2}=20.877$$
$$c=\sqrt{a^2+b^2+h^2}=\sqrt{4.225^2+20.445^2+9^2}=22.734$$

$$Sc_{sIV} = \frac{120}{2\pi\,14000}\left\{\left(20.445\ln\frac{(22.734-4.225)\,(20.877+4.225)}{(22.734+4.255)\,(20.877-4.225)}+\right.\right.$$
$$\left.\left.+4.225\ln\frac{(22.734-20.445)\,(20.877+20.445)}{(22.734+20.445)\,(20.877-20.445)}++9\tan^{-1}\frac{4.225\times 20.445}{9\times 22.734}\right)\right\}$$
$$Sc_{sIV} = 0.0152[\text{m}]$$

The steps of the calculation of the settlement Sc_s for the sand layer are summarized in Table 4.10.

Table 4.10 Steps of calculation of consolidation settlement Sc_s for the sand layer

Area No.	Side *a* [m]	Side *b* [m]	$m=\sqrt{a^2+b^2}$ [m]	$c=\sqrt{a^2+b^2+h^2}$ [m]	Settlement Sc_{si} [m]
I	28.275	3.055	28.440	29.830	0.0135
II	4.225	3.055	5.214	10.401	0.0117
III	28.275	20.445	34.892	36.034	0.0191
IV	4.225	20.445	20.877	22.734	0.0152
Total settlement of the sand layer Sc_s =Σ					0.0595

4.10.5.2.2 Settlement of the clay layer

4.10.5.2.3 Preconsolidation pressure σ_c

In this example the over consolidated ratio is given *OCR* =1.5. Therefore, preconsolidation pressure of the clay σ_c is estimated from:

$$\sigma_c = OCR \times \sigma_o$$

To determine the depth under the surface in which the clay is preconsolidation for reloading case ($\sigma_f < \sigma_c$) or loading and reloading case ($\sigma_f > \sigma_c > \sigma_o$), a comparison between preconsolidation pressure σ_c and total vertical stress on the clay $\sigma_f = \sigma_o + \sigma_z$. is carried out as shown in Table 4.11. To achieve that, the total clay layer is subdivided into sub-layers, each of 1.0 [m] thickness. In the middle of each sub-layer, the overburden pressure σ_o, vertical stress σ_z, preconsolidation pressure σ_c and final stress on the sub-layer σ_f are determined. Because of the sub-layer thickness is very small, the increment of vertical stress in the sub-layer $\Delta\sigma_{av}$ is considered equal to the vertical stress σ_z in the middle of the sub-layer. Consequently, the consolidation settlement Sc_c can be determined in the same time.

4.10.5.2.4 Vertical stress in sub layers

Stress σ_z at a depth *z* in the soil under a corner of a rectangular loaded area *q* with sides *a* and *b* at the surface is given by:

$$\sigma_z = \frac{q}{2\pi}\left[\left(\frac{1}{R_1^2} + \frac{1}{R_2^2}\right)\frac{a.b.z}{R_3} + \tan^{-1}\frac{a.b}{z.R_3}\right]$$

where $R_1^2 = \sqrt{a^2 + z^2}$, $R_2^2 = \sqrt{b^2 + z^2}$ and $R_3 = \sqrt{a^2 + b^2 + z^2}$

4.10.5.2.5 Overburden pressure

The overburden pressure at the surface of the clay layer is given by:

$$\sigma_o = \gamma'_s d_f + \gamma'_s h_s = 16 \times 1.5 + 7 \times 9 = 87 \text{ [kN/m}^2\text{]}$$

The overburden pressure at any depth *z* under the raft in the clay layer is given by:

$$\sigma_o = 87 + \gamma'_c (z - h_s) = 33 + 6z$$

The steps of the calculation of the consolidation settlement Sc_c for the clay layer are given in Table 4.11.

Table 4.11 Steps of calculation of consolidation settlement Sc_c for the clay layer

Sub layer thickness hc [m]	Depth under the raft z [m]	σ_o = 33+6z [kN/m^2]	σ_c = $OCR\times\sigma_o$ 1.5 σ_o [kN/m^2]	σ_z [kN/m^2]	σ_f =σ_o+σ_z [kN/m^2]	Settlement Sc_{ci} [m]
hc1=3 [m]	$S_c = \frac{C_r\,h}{1+e_o}\log\left(\frac{\sigma_c}{\sigma_o}\right) + \frac{C_c\,h}{1+e_o}\log\left(\frac{\sigma_o+\Delta\sigma_{av}}{\sigma_c}\right)$ where $\sigma_f > \sigma_c > \sigma_o$					
	9.5	90	135	61.61	151.61	0.0257
	10.5	96	144	58.04	154.04	0.0193
	11.5	102	153	54.91	156.91	0.0139
	Total consolidation settlement Sc_{c1} =Σ					0.0589
hc2=11 [m]	$Sc_{c2} = \frac{C_r\,h}{1+e_o}\log\left(\frac{\sigma_o+\Delta\sigma_{va1}}{\sigma_o}\right)$ where $\sigma_f < \sigma_c$					
	12.5	108	162	52.12	160.12	0.0103
	13.5	114	171	49.61	163.61	0.0094
	14.5	120	180	47.33	167.33	0.0087
	15.5	126	189	45.22	171.22	0.0080
	16.5	132	198	43.27	175.27	0.0074
	17.5	138	207	41.45	179.45	0.0068
	18.5	144	216	39.75	183.75	0.0064
	19.5	150	225	38.14	188.14	0.0059
	20.5	156	234	36.62	192.62	0.0055
	21.5	162	243	35.18	197.18	0.0051
	22.5	168	252	33.81	201.81	0.0048
	Total consolidation settlement of the clay layer Sc_{c2} =Σ					0.0782

4.10.5.2.6 Preconsolidation settlement:

The total settlement of the raft occurred due to two parts. Preconsolidation part for $\sigma_f < \sigma_c$, and preconsolidation part for $\sigma_f > \sigma_c > \sigma_o$. This settlement can be obtained from the following equations:

For a preconsolidated clay, if $\sigma_f < \sigma_c$ (reloading case), Figure 4.7:

$$S_c = \frac{C_r\,h}{1+e_o}\log\left(\frac{\sigma_o+\Delta\sigma_{av}}{\sigma_o}\right) \tag{43}$$

For a preconsolidated clay, if $\sigma_f > \sigma_c > \sigma_o$ (loading and reloading case), Figure 4.8:

$$S_c = \frac{C_r\,h}{1+e_o}\log\left(\frac{\sigma_c}{\sigma_o}\right) + \frac{C_c\,h}{1+e_o}\log\left(\frac{\sigma_o + \Delta\sigma_{av}}{\sigma_c}\right) \qquad (44)$$

where:

σ_o Initial overburden pressure at the middle of the layer, $\sigma_o = \gamma z$, [kN/m^2].
$\Delta\sigma_{av}$ Average increase of stress in the layer due to the applied load, [kN/m^2].
σ_c Preconsolidation pressure of the layer, [kN/m^2].
C_c Compression index for loading, [-].
C_r Compression index for reloading, [-].
e_o Initial void ratio, [-].

The preconsolidation settlement is then:
For $\sigma_f > \sigma_c > \sigma_o$ (loading and reloading case) Sc_{c1} = 0.0589 [m]
For $\sigma_f < \sigma_c$ (reloading case) Sc_{c2} = 0.0782 [m]
Total preconsolidation settlement is Sc_c = 0.1371 [cm]

4.10.5.3 Settlement by GEO Tools

GEO Tools deals with the clay layer as one unit and considers the different stresses for the whole layer. Therefore, to ensure the variable cases of preconsolidation of the clay, the clay layer must be subdivided into sub-layers. In this example, the total clay layer is subdivided into two layers. The settlement obtained from *GEO Tools* at the characteristic point *o* of the raft is 19.59 [cm] nearly the same as that of the hand calculation with a difference 0.07 [cm]. Table 4.12 compares between settlement obtained from *GEO Tools* and that of the hand calculation at the characteristic point *o* of the raft. The input data and results of *GEO Tools* are presented on the next pages.

Table 4.12 Settlement at the characteristic point *o* of the raft

Layer	Layer thickness [m]	Consolidation settlement [cm]		Difference [cm]
		Hand calculation $S_{c(i)}$	*GEO Tools* $S_{c(i)}$	
Sand	9	5.95	5.95	
Clay	3	5.89	5.84	
Clay	11	7.82	7.81	
Total settlement $Sc = \Sigma$		19.66	19.59	0.07

```
**************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
**************************************************************
Title: A tower of 10 floors
Date: 02-09-2017
Project: Settlement calculation
File: 4Con_Ex4

--------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
--------------------------------------------------------------

Data:
Rectangular load                      q       [kN/m2] = 120.0
Length                                a       [m]     = 32.50
Width                                 b       [m]     = 23.50

Soil Data:
Layer No.: 1
Modulus of compressibility            Es      [kN/m2] = 14000
Unit weight                           Gamma_c [kN/m3] = 7
Layer thickness                       h       [m]     = 9.00

Layer No.: 2
Compression index                     Cc      [-]     = 0.750
Recompression Index                   Cr      [-]     = 0.150
Over consolidated ratio               OCR     [-]     = 1.500
Initial void ratio                    eo      [-]     = 1.500
Unit weight                           Gamma_c [kN/m3] = 6
Layer thickness                       h       [m]     = 3.00

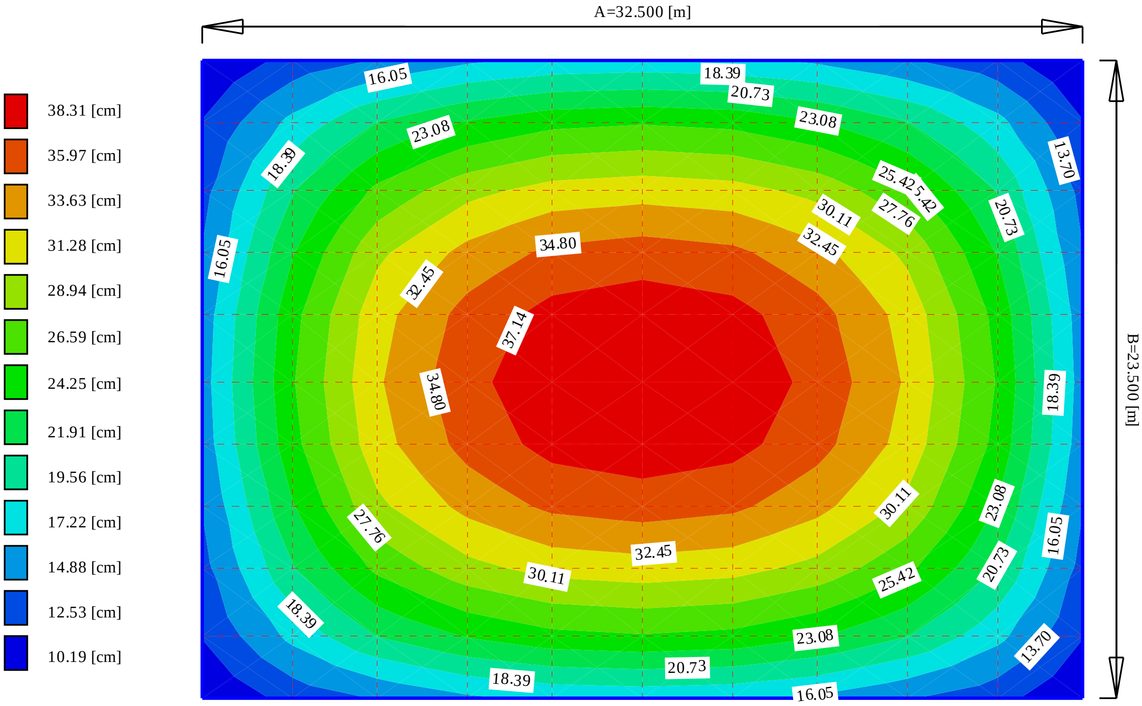
Layer No.: 3
Compression index                     Cc      [-]     = 0.750
Recompression Index                   Cr      [-]     = 0.150
Over consolidated ratio               OCR     [-]     = 1.500
Initial void ratio                    eo      [-]     = 1.500
Unit weight                           Gamma_c [kN/m3] = 6
Layer thickness                       h       [m]     = 11.00


Overburden pressure                   Gamma*z [kN/m2] = 24.00
Depth increment in z-direction        Dz      [m]     = 0.00

Point coordinates:
X-coord.                              x       [m]     = 4.22
Y-coord.                              y       [m]     = 3.05
Y-coord.                              z       [m]     = 0.00

Results:
Settlement
Layer No.: 1                          S1      [cm]    = 5.95
Layer No.: 2                          S2      [cm]    = 5.84
Layer No.: 3                          S3      [cm]    = 7.81
Total                                 St      [cm]    = 19.59
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 39.48 at node 61, Min. s = 9.01 at node 1

GEOTEC Software Inc., Calgary AB, Canada T3E 7Y7 PO Box 14001 Richmond Road PO, Tele.: +1 (587) 332-3323	
Scale: 208	**Project: Settlement calculation**
File: 4Con_Ex4	**Date: 02-09-2017**
Page No.:	**Title: A tower of 10 floors**

4.10.6 Example 5: Settlement of different soil layers

4.10.6.1 Description of the problem

To verify the settlement of different soil layers calculated by the program *GEO Tools*, a hand calculation of a settlement for three different soil layers under three types of loading is compared with that obtained by *GEO Tools*.

The soil profile consists of three different soil layers as shown in 0. Properties of the three soil layers are shown in Table 4.13. For the three types of loading shown in Figure 4.29, find the total settlement of the three layers at the:

a) center of a circular loaded area of q = 100 [kN/m^2] and a radius a = 5 [m].
b) center of a rectangular loaded area of q = 100 [kN/m^2] and sides A×B=6 [m]×4 [m].
c) distance x=1 [m] from a point load of Q = 3000 [kN].

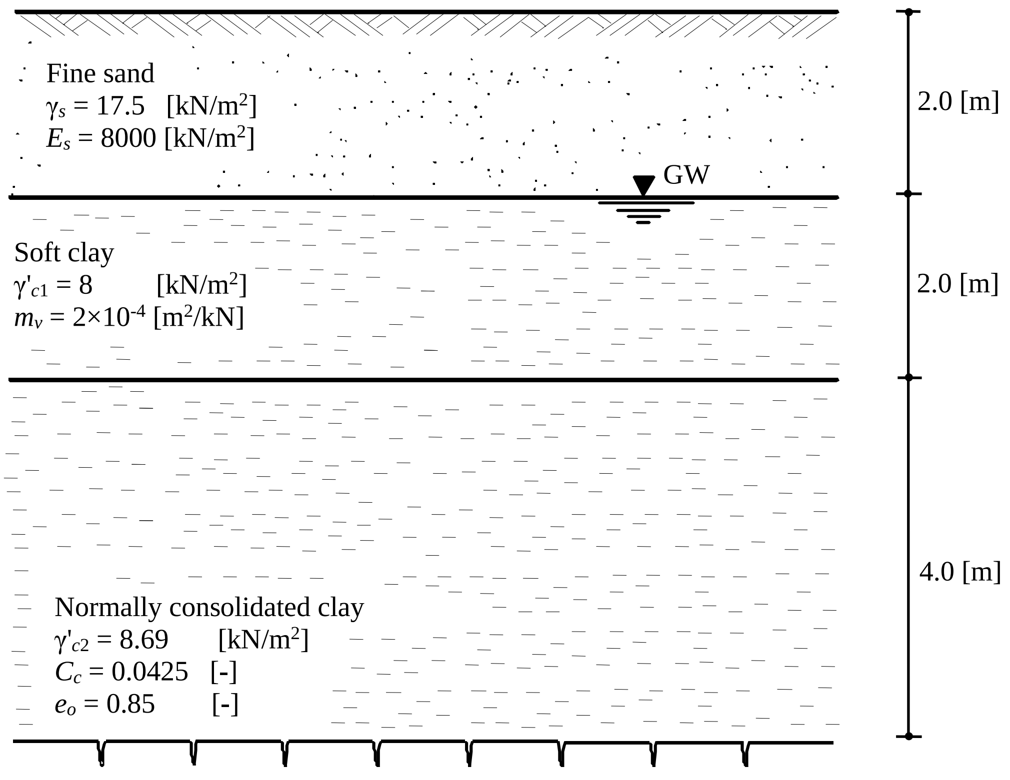


Figure 4.28 Soil profile

Table 4.13 Properties of the soil layers

Layer No.	Layer type	Thickness h [m]	Unite weight of the soil γ [kN/m^3]	Soil properties
1	Sand	2	17.5	E_s=8000 [kN/m^2]
2	Clay	2	8.0	m_v=2×10^{-4} [m^2/kN]
3	Clay	4	8.69	e_o=0.85 [-] C_c=0.0425 [-]

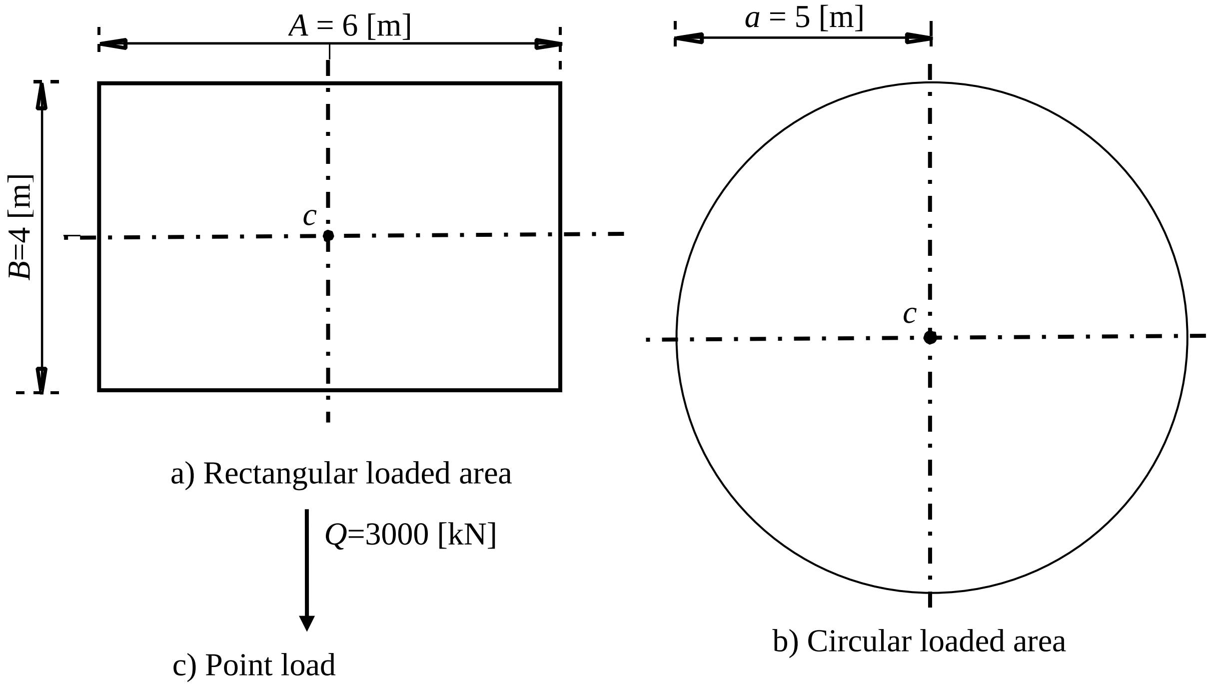


Figure 4.29 Loading types

4.10.6.2 Hand calculation

GEO Tools deals with the clay layer as one unit and considers the stress for the whole layer. Here, for simplifying the solution by the hand calculation, the stress is calculated at the middle of each layer in the cases of loaded areas. For the point load, the stress is calculated in the entire layer.

4.10.6.2.1 Settlement under the center of the rectangular loaded area

Stress σ_z at a depth z in the soil under the center of a rectangular loaded area of sides $2a \times 2b$ is given by:

$$\sigma_z = \frac{4q}{2\pi}\left[\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}\right)\frac{a.b.z}{R_3}+\tan^{-1}\frac{a.b}{z.R_3}\right]$$

where:

$$R_1^2 = \sqrt{a^2+z^2}$$

$$R_2^2 = \sqrt{b^2+z^2}$$

$$R_3 = \sqrt{a^2+b^2+z^2}$$

Stress σ_z at the middle of the first layer:

$$R_1^2 = 3^2 + 1^2 = 10$$
$$R_2^2 = 2^2 + 1^2 = 5$$
$$R_3 = \sqrt{3^2 + 2^2 + 1^2} = 3.74$$
$$\sigma_1 = \frac{4\times 100}{2\pi}\left[\left(\frac{1}{10} + \frac{1}{5}\right)\frac{3\times 2\times 1}{3.74} + \tan^{-1}\frac{3\times 2}{1\times 3.74}\right] = 95.15 \text{ [kN/m}^2\text{]}$$

Stress σ_z at the middle of the second layer:

$$R_1^2 = 3^2 + 3^2 = 18$$
$$R_2^2 = 2^2 + 3^2 = 13$$
$$R_3 = \sqrt{3^2 + 2^2 + 3^2} = 4.69$$
$$\sigma_1 = \frac{4\times 100}{2\pi}\left[\left(\frac{1}{18} + \frac{1}{13}\right)\frac{3\times 2\times 3}{4.69} + \tan^{-1}\frac{3\times 2}{3\times 4.69}\right] = 58.03 \text{ [kN/m}^2\text{]}$$

Stress σ_z at the middle of the third layer:

$$R_1^2 = 3^2 + 6^2 = 45$$
$$R_2^2 = 2^2 + 6^2 = 40$$
$$R_3 = \sqrt{3^2 + 2^2 + 6^2} = 7$$
$$\sigma_1 = \frac{4\times 100}{2\pi}\left[\left(\frac{1}{45} + \frac{1}{40}\right)\frac{3\times 2\times 6}{7} + \tan^{-1}\frac{3\times 2}{6\times 7}\right] = 24.49 \text{ [kN/m}^2\text{]}$$

Overburden stress σ_o at the middle of the third layer:

$$\sigma_o = 2\times 17.5 + 2\times 8 + 2\times 8.69 = 68.38 \text{ [kN/m}^2\text{]}$$

Settlement *s* of the first layer:

$$s_1 = \frac{1}{E_s}\Delta\sigma h = \frac{1}{8000}\times 95.15\times 2 = 0.0248 \text{ [m]} = 2.48 \text{ [cm]}$$

Settlement *s* of the second layer:

$$s_2 = m_v \Delta\sigma h = 2\times 10^{-4}\times 58.03\times 2 = 0.0232 \text{ [m]} = 2.32 \text{ [cm]}$$

Settlement *s* of the third layer:

$$s_3 = \frac{C_c h}{1+e_o}\log\frac{\Delta\sigma+\sigma_o}{\sigma_o} = \frac{0.0425\times 4}{1+0.85}\log\frac{24.49+68.38}{68.38} = 0.0122 \text{ [m]} = 1.22 \text{ [cm]}$$

Total settlement s of all layers:

$$s_t = s_1 + s_2 + s_3 = 2.48 + 2.32 + 1.22 = 6.02 \text{ [cm]}$$

4.10.6.2.2 Settlement under the center of the circular loaded area

Stress σ_z at a depth z in the soil under the center of a circular loaded area of radius a is given by:

$$\sigma_z = q\left[1 - \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$

Stress σ_z at the middle of the first layer (z=1 [m]):

$$\sigma_1 = 100\left[1 - \frac{1^3}{(25+1^2)^{3/2}}\right] = 99.25 \text{ [kN/m}^2\text{]}$$

Stress σ_z at the middle of the second layer (z=3 [m]):

$$\sigma_2 = 100\left[1 - \frac{3^3}{(25+9)^{3/2}}\right] = 86.38 \text{ [kN/m}^2\text{]}$$

Stress σ_z at the middle of the third layer (z=6 [m]):

$$\sigma_3 = 100\left[1 - \frac{6^3}{(25+36)^{3/2}}\right] = 54.66 \text{ [kN/m}^2\text{]}$$

Overburden stress σ_o at the middle of the third layer:

$$\sigma_o = 2\times 17.5 + 2\times 8 + 2\times 8.69 = 68.38 \text{ [kN/m}^2\text{]}$$

Settlement s of the first layer:

$$s_1 = \frac{1}{E_s}\Delta\sigma h = \frac{1}{8000}\times 99.25\times 2 = 0.0248 \text{ [m]} = 2.48 \text{ [cm]}$$

Settlement s of the second layer:

$$s_2 = m_v\Delta\sigma h = 2\times 10^{-4}\times 86.38\times 2 = 0.0346 \text{ [m]} = 3.46 \text{ [cm]}$$

Settlement s of the third layer:

$$s_3 = \frac{C_c h}{1+e_o}\log\frac{\Delta\sigma+\sigma_o}{\sigma_o} = \frac{0.0425\times 4}{1+0.85}\log\frac{54.66+68.38}{68.38} = 0.0234\,[\text{m}] = 2.34\ [\text{cm}]$$

Total settlement s of all layers:

$$s_t = s_1 + s_2 + s_3 = 2.48 + 3.46 + 2.34 = 8.28\ [\text{cm}]$$

4.10.6.2.3 Settlement under the point load

The average stress $\Delta\sigma_{va}$ in a soil layer of thickness h from depth h_1 to h_2 under a point load Q is given by:

$$\Delta\sigma_{va} = \frac{Q}{2\pi\, h}\left(\frac{r^2}{\left(h_2^2+r^2\right)^{\frac{3}{2}}} - \frac{3}{\sqrt{h_2^2+r^2}} - \frac{r^2}{\left(h_1^2+r^2\right)^{\frac{3}{2}}} + \frac{3}{\sqrt{h_1^2+r^2}}\right)$$

Average stress $\Delta\sigma_{va}$ in the first soil layer:

$$\Delta\sigma_{va} = \frac{3000}{2\pi\, 2}\left(\frac{1^2}{\left(2^2+1^2\right)^{\frac{3}{2}}} - \frac{3}{\sqrt{2^2+1^2}} - \frac{1^2}{\left(0+1^2\right)^{\frac{3}{2}}} + \frac{3}{\sqrt{0+1^2}}\right) = 178.52\,[\text{kN/m}^2]$$

Average stress $\Delta\sigma_{va}$ in the second soil layer:

$$\Delta\sigma_{va} = \frac{3000}{2\pi\, 2}\left(\frac{1^2}{\left(4^2+1^2\right)^{\frac{3}{2}}} - \frac{3}{\sqrt{4^2+1^2}} - \frac{1^2}{\left(2^2+1^2\right)^{\frac{3}{2}}} + \frac{3}{\sqrt{2^2+1^2}}\right) = 128.64\,[\text{kN/m}^2]$$

Average stress $\Delta\sigma_{va}$ in the third soil layer:

$$\Delta\sigma_{va} = \frac{3000}{2\pi\, 4}\left(\frac{1^2}{\left(8^2+1^2\right)^{\frac{3}{2}}} - \frac{3}{\sqrt{8^2+1^2}} - \frac{1^2}{\left(4^2+1^2\right)^{\frac{3}{2}}} + \frac{3}{\sqrt{4^2+1^2}}\right) = 40.96\,[\text{kN/m}^2]$$

Overburden stress σ_o at the middle of the third layer:

$$\sigma_o = 2\times 17.5 + 2\times 8 + 2\times 8.69 = 68.38\ [\text{kN/m}^2]$$

Settlement s of the first layer:

$$s_1 = \frac{1}{E_s}\Delta\sigma h = \frac{1}{8000}\times 178.52\times 2 = 0.0446\ [\text{m}] = 4.46\ [\text{cm}]$$

Settlement s of the second layer:

$$s_2 = m_v \Delta\sigma h = 2\times10^{-4}\times128.64\times2 = 0.0515\ \text{[m]} = 5.15\,\text{[cm]}$$

Settlement *s* of the third layer:

$$s_3 = \frac{C_c h}{1+e_o}\log\frac{\Delta\sigma+\sigma_o}{\sigma_o} = \frac{0.0425\times4}{1+0.85}\log\frac{40.96+68.38}{68.38} = 0.0187\,\text{[m]} = 1.87\,\text{[cm]}$$

Total settlement *s* of all layers:

$$s_t = s_1 + s_2 + s_3 = 4.46+5.15+1.87 = 11.48\ \text{[cm]}$$

4.10.6.3 Settlement by GEO Tools

The exact settlements obtained from *GEO Tools* for the three types of loading are compared with those obtained by the hand calculation in Table 4.14 to 00. The settlements obtained from *GEO Tools* at the center *c* of the rectangular loaded area is 5.98 [cm], while that at the center of the circular loaded area is 8.32 [cm]. The exact settlement obtained from *GEO Tools* under the point load is 11.51 [cm]. They are nearly same as those of the hand calculations with a difference between 0.03 to 0.04 [cm]. The input data and results of *GEO Tools* are presented on the next pages.

Table 4.14 Settlement at the center of the rectangular loaded area

Layer	Layer thickness [m]	Consolidation settlement [cm]		Difference [cm]
		Hand calculation $S_{c(i)}$	*GEO Tools* $S_{c(i)}$	
Sand	2	2.48	2.32	
Clay	2	2.32	2.35	
Clay	4	1.22	1.31	
Total settlement $Sc = \Sigma$		6.02	5.98	0.04

Table 4.15 Settlement at the center of the circular loaded area

Layer	Layer thickness [m]	Consolidation settlement [cm]		Difference [cm]
		Hand calculation $S_{c(i)}$	*GEO Tools* $S_{c(i)}$	
Sand	2	2.48	2.47	
Clay	2	3.46	3.44	
Clay	4	2.34	2.41	
Total settlement $Sc = \Sigma$		8.28	8.32	0.04

Table 4.16 Settlement under the point load (stress is calculated a in the entire layer)

Layer	Layer thickness [m]	Consolidation settlement [cm]		Difference [cm]
		Hand calculation $S_{c(i)}$	*GEO Tools* $S_{c(i)}$	
Sand	2	4.46	4.46	
Clay	2	5.15	5.15	
Clay	4	1.87	1.90	
Total settlement $Sc = \Sigma$		11.48	11.51	0.03

```
************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Settlement of different soil layers
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex5

------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
------------------------------------------------------------

Data:
Rectangular load                     q        [kN/m2] = 100.0000
Length                               a        [m]     = 6
Width                                b        [m]     = 4

Soil Data:
Layer No.: 1
Modulus of compressibility           Es       [kN/m2] = 8000.0000
Unit weight                          Gamma_c  [kN/m3] = 18.00000
Layer thickness                      h        [m]     = 2

Layer No.: 2
Modulus of compressibility           Es       [kN/m2] = 5000.0000
Unit weight                          Gamma_c  [kN/m3] = 8.00000
Layer thickness                      h        [m]     = 2

Layer No.: 3
Compression index                    Cc       [-]     = 0.043
Recompression Index                  Cr       [-]     = 0.043
Over consolidated ratio              OCR      [-]     = 1.000
Initial void ratio                   eo       [-]     = 0.850
Unit weight                          Gamma_c  [kN/m3] = 8.00000
Layer thickness                      h        [m]     = 4

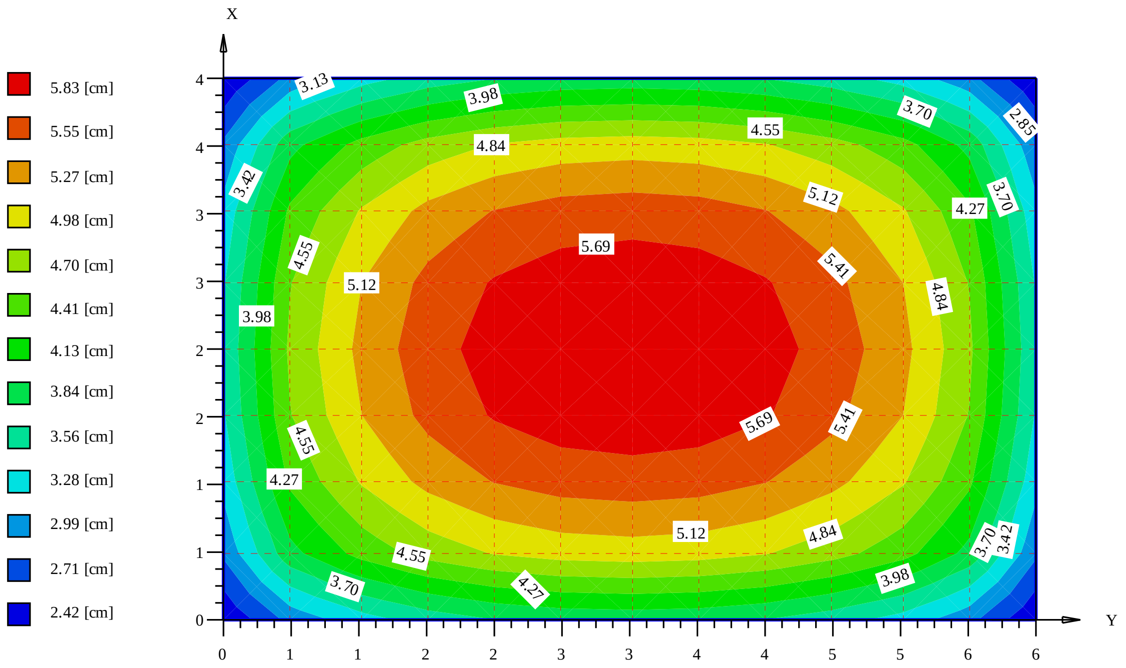

Overburden pressure                  Gamma*z  [kN/m2] = 0.0000


Point coordinates:
X-coord.                             x        [m]     = 3
Y-coord.                             y        [m]     = 2
Y-coord.                             z        [m]     = 0

Results:
Settlement
Layer No.: 1                         S1       [cm]    = 2.32
Layer No.: 2                         S2       [cm]    = 2.35
```

```
Layer No.: 3                                S3      [cm]     = 1.31
Total                                       St      [cm]     = 5.98
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 5.98 at node 59, Min. s = 2.28 at node 1

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PO Box 14001 Richmond Road PO, Tele.: +1 (587) 332-3323

Scale: 41	**Project: User's Manual**
File: 4Con_Ex5	**Date: 01-09-2017**
Page No.:	**Title: Settlement of different soil layers**

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************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Settlement of different soil layers
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex5

------------------------------------------------------------
Consolidation settlement in soil due to circular load
------------------------------------------------------------

Data:
Circular load                        p        [kN/m2] = 100.0000
Radius                               a        [m]     = 5

Soil Data:
Layer No.: 1
Modulus of compressibility           Es       [kN/m2] = 8000.0000
Unit weight                          Gamma_c  [kN/m3] = 18.00000
Layer thickness                      h        [m]     = 2

Layer No.: 2
Modulus of compressibility           Es       [kN/m2] = 5000.0000
Unit weight                          Gamma_c  [kN/m3] = 8.00000
Layer thickness                      h        [m]     = 2

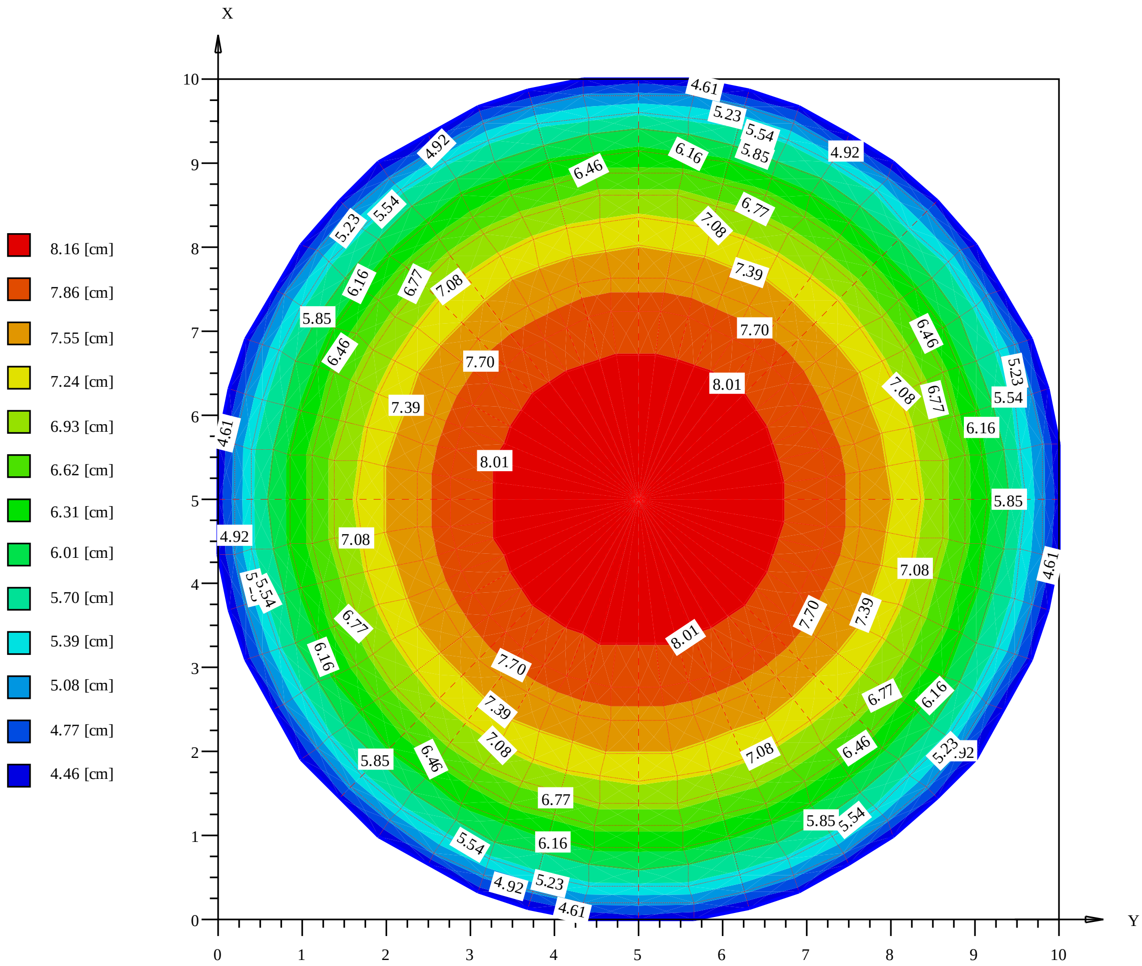
Layer No.: 3
Compression index                    Cc       [-]     = 0.043
Recompression Index                  Cr       [-]     = 0.043
Over consolidated ratio              OCR      [-]     = 1.000
Initial void ratio                   eo       [-]     = 0.850
Unit weight                          Gamma_c  [kN/m3] = 8.00000
Layer thickness                      h        [m]     = 4


Overburden pressure                  Gamma*z  [kN/m2] = 0.0000


Point coordinates:
X-coord.                             x        [m]     = 5
Y-coord.                             y        [m]     = 5
Y-coord.                             z        [m]     = 0

Results:
Settlement
Layer No.: 1                         S1       [cm]    = 2.47
Layer No.: 2                         S2       [cm]    = 3.44
Layer No.: 3                         S3       [cm]    = 2.41
Total                                St       [cm]    = 8.32
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 8.32 at node 1, Min. s = 4.30 at node 493

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Scale: 64	**Project: User's Manual**
File: 4Con_Ex5	**Date: 01-09-2017**
Page No.:	**Title: Settlement of different soil layers**

```
*************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
*************************************************************
Title: Settlement of different soil layers
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex5

-------------------------------------------------------------
Consolidation settlement in soil due to point load
-------------------------------------------------------------

Data:
Point load                           Q        [kN]    = 3000.00

Soil Data:
Layer No.: 1
Modulus of compressibility           Es       [kN/m2] = 8000.0000
Unit weight                          Gamma_c  [kN/m3] = 18.00000
Layer thickness                      h        [m]     = 2

Layer No.: 2
Modulus of compressibility           Es       [kN/m2] = 5000.0000
Unit weight                          Gamma_c  [kN/m3] = 8.00000
Layer thickness                      h        [m]     = 2

Layer No.: 3
Compression index                    Cc       [-]     = 0.043
Recompression Index                  Cr       [-]     = 0.043
Over consolidated ratio              OCR      [-]     = 1.000
Initial void ratio                   eo       [-]     = 0.850
Unit weight                          Gamma_c  [kN/m3] = 8.00000
Layer thickness                      h        [m]     = 4


Overburden pressure                  Gamma*z  [kN/m2] = 0.0000

Point coordinates:
X-coord.                             x        [m]     = 1
Y-coord.                             y        [m]     = 0
Y-coord.                             z        [m]     = 0

Results:
Settlement
Layer No.: 1                         S1       [cm]    = 4.46
Layer No.: 2                         S2       [cm]    = 5.15
Layer No.: 3                         S3       [cm]    = 1.90
Total                                St       [cm]    = 11.51
```

4.10.7 Example 6: Settlement of a circular loaded area resting on a thin clay layer

4.10.7.1 Description of the problem

To verify the settlement of a loaded area resting on a relative thin clay layer calculated by the program *GEO Tools*, a hand calculation of a settlement for a relative thin soil layer under a circular loaded area is compared with that obtained by *GEO Tools*.

A circular loaded area of a load q = 150 [kN/m^2] and radius a = 4 [m] is acting on a relative thin clay layer as shown in Figure 4.30. Find the settlement of the clay layer under the center of the loaded area.

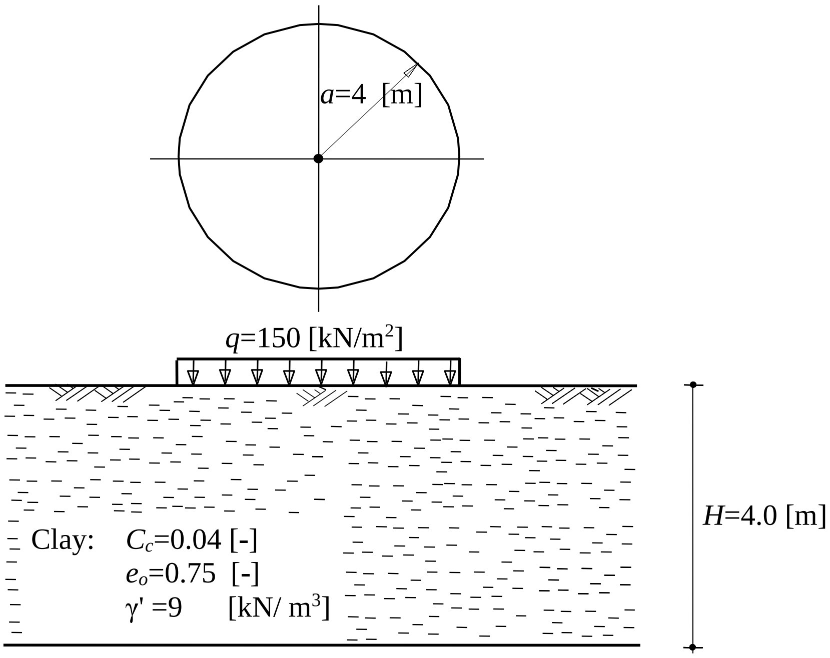


Figure 4.30 Soil profile under the circular loaded area

4.10.7.1 Hand calculation

If the clay layer is relatively thin and its thickness doesn't exceed on the footing length H<2a, *GEO Tools* deals with the clay layer as one unit and considers the stress for the whole layer. The average stress $\Delta\sigma_{va}$ in a relative thin clay layer of thickness H under the center of a circular loaded area q of a radius a is given by:

$$\Delta\sigma_{va} = \frac{q}{H}\left(H - \frac{H^2 + 2a^2}{\sqrt{H^2 + a^2}} + 2a \right)$$

The average stress $\Delta\sigma_{va}$ in the entire clay layer:

$$\Delta\sigma_{va} = \frac{150}{4}\left(4 - \frac{4^2 + 2\times 4^2}{\sqrt{4^2 + 4^2}} + 2\times 4\right)$$

$$\Delta\sigma_{va} = 131.8\,[\text{kN/m}^2]$$

Overburden stress σ_o at the middle of the clay layer (z=2 [m]):

$$\sigma_o = \gamma' z_2 = 9\times 2 = 18\ [\text{kN/m}^2]$$

Settlement S_c of the clay layer:

$$S_c = \frac{C_c H}{1+e_o}\log\frac{\Delta\sigma_{va}+\sigma_o}{\sigma_o} = \frac{0.04\times 4}{1+0.75}\log\frac{131.8+18}{18} = 0.0841\ [\text{m}] = 8.41[\text{cm}]$$

4.10.7.2 Settlement by GEO Tools

The settlement obtained from *GEO Tools* at the center *c* of the circular loaded area is 8.41 [cm]. It is exactly same as that of the hand calculation. The input data and results of *GEO Tools* are presented on the next pages.

```
**************************************************************
                         GEO Tools
                          Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
**************************************************************
Title: Settlement of a circular loaded area resting on a thin clay layer
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex6

--------------------------------------------------------------
Consolidation settlement in soil due to circular load
--------------------------------------------------------------

Data:
Circular load                         p        [kN/m2] = 150.0000
Radius                                a        [m]     = 4.00

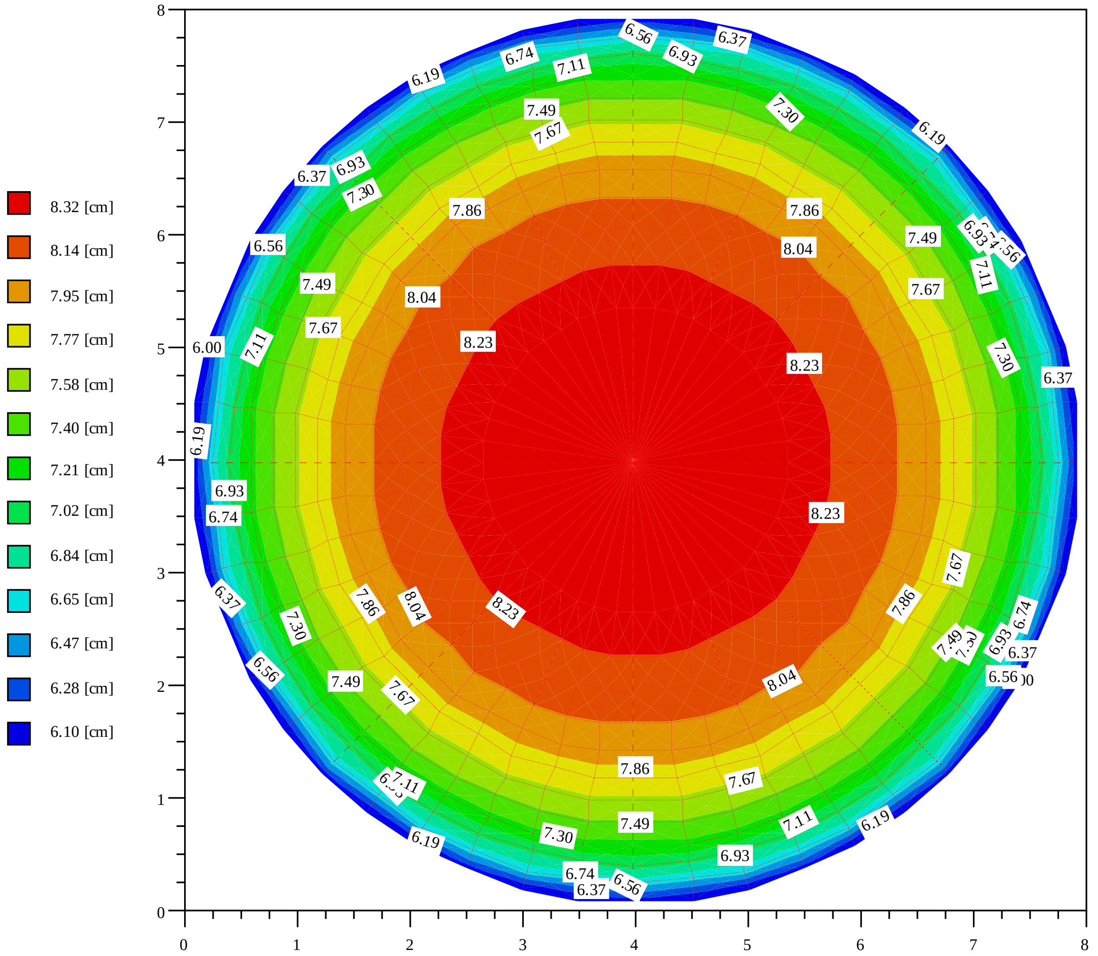
Soil Data:
Layer No.: 1
Compression index                     Cc       [-]     = 0.040
Recompression Index                   Cr       [-]     = 0.040
Over consolidated ratio               OCR      [-]     = 1.000
Initial void ratio                    eo       [-]     = 0.750
Unit weight                           Gamma_c  [kN/m3] = 9.00000
Layer thickness                       h        [m]     = 4.00


Overburden pressure                   Gamma*z  [kN/m2] = 0.0000
Depth increment in z-direction        Dz       [m]     = 0.50

Point coordinates:
X-coord.                              x        [m]     = 4.00
Y-coord.                              y        [m]     = 4.00
Y-coord.                              z        [m]     = 0.00

Results:
Settlement
Layer No.: 1                          S1       [cm]    = 8.41
Total                                 St       [cm]    = 8.41
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 8.41 at node 1, Min. s = 6.00 at node 25

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Scale: 49	**Project: User's Manual**
File: 4Con_Ex6	**Date: 01-09-2017**
Page No.:	**Title: Settlement of a circular loaded area resting on a clay layer**

4.10.8 Example 7: Settlement of a circular loaded area resting on a thick clay layer

4.10.8.1 Description of the problem

To verify the settlement of a loaded area resting on a thick clay layer calculated by the program *GEO Tools*, a hand calculation of a settlement for a thick soil layer under a circular loaded area is compared with that obtained by *GEO Tools*.

A circular loaded area of a load q = 150 [kN/m^2] and radius a = 4 [m] is acting on a thick clay layer as shown in Figure 4.31. Find the settlement of the clay layer under the center of the loaded area.

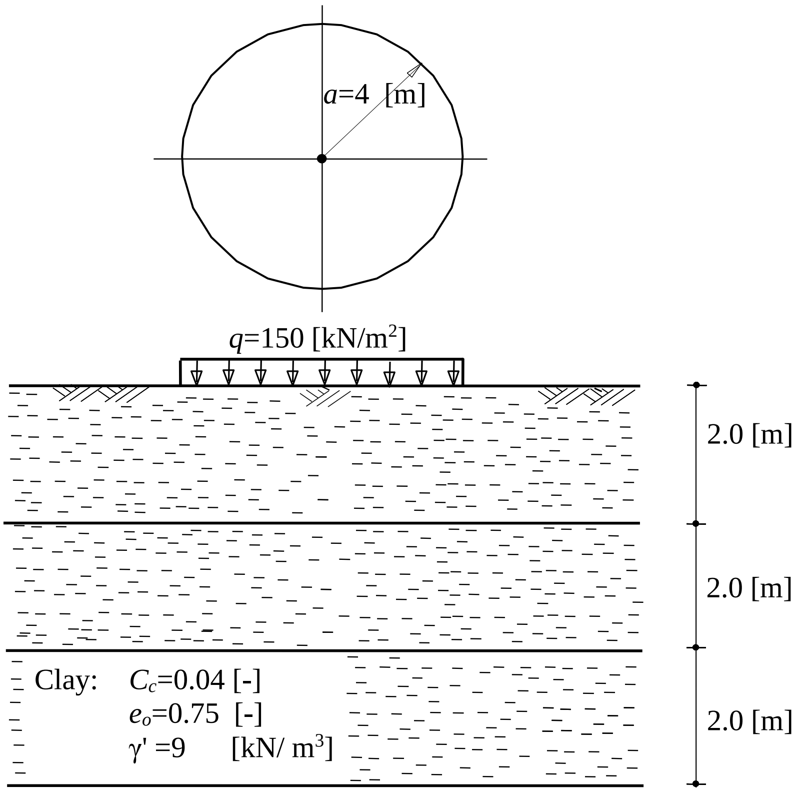


Figure 4.31 Soil profile under the circular loaded area

4.10.8.2 Hand calculation

GEO Tools subdivides the thick clay layer into sub-layers, then the average stress in each sub-layer is determined. Here, for simplifying the solution by the hand calculation, the stress is calculated at the middle of each sub-layer.

Stress σ_z at a depth z in the soil under the center of a circular loaded area q of radius a is given by:

$$\sigma_z = q\left[1 - \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$

Stress σ_z at the middle of the first sub layer (z=1 [m]):

$$\sigma_1 = 150\left[1 - \frac{1^3}{(16+1^2)^{3/2}}\right] = 147.86 \text{ [kN/m}^2\text{]}$$

Stress σ_z at the middle of the second sub layer (z=3 [m]):

$$\sigma_2 = 150\left[1 - \frac{3^3}{(16+9)^{3/2}}\right] = 117.6 \text{ [kN/m}^2\text{]}$$

Stress σ_z at the middle of the third sub layer (z=5 [m]):

$$\sigma_3 = 150\left[1 - \frac{5^3}{(16+25)^{3/2}}\right] = 78.58 \text{ [kN/m}^2\text{]}$$

Overburden stress σ_o at the middle of the first sub layer (z=1 [m]):

$$\sigma_o = \gamma' z_1 = 9 \times 1 = 9 \text{ [kN/m}^2\text{]}$$

Overburden stress σ_o at the middle of the second sub layer (z=3 [m]):

$$\sigma_o = \gamma' z_2 = 9 \times 3 = 27 \text{ [kN/m}^2\text{]}$$

Overburden stress σ_o at the middle of the third sub layer (z=5 [m]):

$$\sigma_o = \gamma' z_3 = 9 \times 5 = 45 \text{ [kN/m}^2\text{]}$$

Settlement S_c of the first sub layer:

$$s_3 = \frac{C_c h}{1+e_o}\log\frac{\Delta\sigma + \sigma_o}{\sigma_o} = \frac{0.04 \times 2}{1+0.75}\log\frac{147.86+9}{9} = 0.0567 \text{ [m]} = 5.67 \text{[cm]}$$

Settlement s of the second sub layer:

$$s_3 = \frac{C_c h}{1+e_o}\log\frac{\Delta\sigma + \sigma_o}{\sigma_o} = \frac{0.04 \times 2}{1+0.75}\log\frac{117.6+27}{27} = 0.0333 \text{ [m]} = 3.33 \text{[cm]}$$

Settlement s of the third sub layer:

$$s_3 = \frac{C_c h}{1+e_o}\log\frac{\Delta\sigma + \sigma_o}{\sigma_o} = \frac{0.04 \times 2}{1+0.75}\log\frac{78.58+45}{45} = 0.02 \text{ [m]} = 2.00 \text{[cm]}$$

Total settlement s of all layers:

$$S_{ct} = s_1 + s_2 + s_3 = 5.67 + 3.33 + 2.00 = 11.01[\text{cm}]$$

4.10.8.3 Settlement by GEO Tools

The exact settlement obtained from *GEO Tools* at the center *c* of the circular loaded area is 10.69 [cm]. It is nearly same as that of the hand calculation with a difference of 0.32 [cm]. The input data and results of *GEO Tools* are presented on the next pages.

```
************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Settlement of a circular loaded area resting on a thick clay layer
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex6

------------------------------------------------------------
Consolidation settlement in soil due to circular load
------------------------------------------------------------

Data:
Circular load                         p        [kN/m2] = 150.0000
Radius                                a        [m]     = 4.00

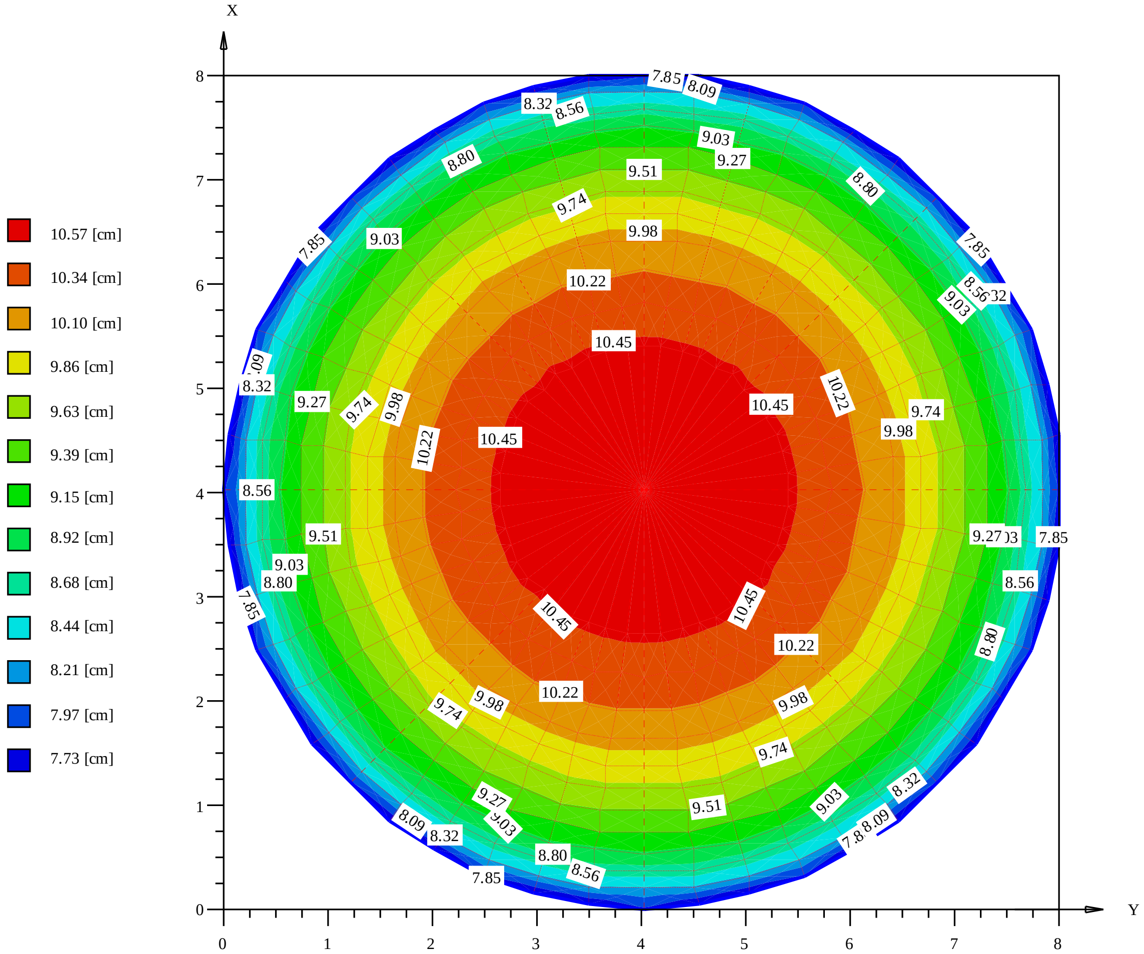
Soil Data:
Layer No.: 1
Compression index                     Cc       [-]     = 0.040
Recompression Index                   Cr       [-]     = 0.040
Over consolidated ratio               OCR      [-]     = 1.000
Initial void ratio                    eo       [-]     = 0.750
Unit weight                           Gamma_c  [kN/m3] = 9.00000
Layer thickness                       h        [m]     = 6.00


Overburden pressure                   Gamma*z  [kN/m2] = 0.0000
Depth increment in z-direction        Dz       [m]     = 0.50

Point coordinates:
X-coord.                              x        [m]     = 4.00
Y-coord.                              y        [m]     = 4.00
Y-coord.                              z        [m]     = 0.00

Results:
Settlement
Layer No.: 1                          S1       [cm]    = 10.69
Total                                 St       [cm]    = 10.69
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 10.69 at node 1, Min. s = 7.61 at node 109

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Scale: 52	**Project: User's Manual**
File: 4Con_Ex7	**Date: 01-09-2017**
Page No.:	**Title: Settlement of a circular loaded area resting on a thick clay layer**

4.10.9 Example 8: Loading and reloading settlement under a rectangular raft

4.10.9.1 Description of the problem

To verify the consolidation settlement calculated by the program *GEO Tools* in the cases of loading and reloading pressures, a hand calculation of a consolidation settlement for a clay layer under a rectangular raft is compared with that obtained by *GEO Tools*.

A rectangular raft of 6 [m] × 4 [m] located at a depth of d_f =2.5 [m] below the ground surface is considered as shown in Figure 4.32. Ground water table lies at a depth z = 2.5 [m] under the ground surface. The contact pressure under the raft is assumed to be uniformly distributed and equal to q = 100 [kN/m^2]. A clay layer of H = 3 [m] thick is located directly under the raft. Figure 4.32 shows raft with dimensions and soil profile, while the soil data are shown Table 4.7Table 4.17.

It is required to determine the final settlement under the center of the raft due to consolidation of the clay for the following three cases of reloading pressure:

a) The reloading pressure is due to the excavation weight, *OCR*=1.65.
Considering compression and recompression indices, C_r=0.02 [-], C_c=0.05 [-].

b) The reloading pressure is due to an old load in the past exceeds to the present applied load, *OCR*=2.5.
Considering a recompression index only, C_r=C_c=0.05 [-].

c) The clay is a virgin clay and the reloading pressure due to the excavation weight is neglected, *OCR*=1.0.
Considering a compression index only, C_c=C_r=0.02 [-].

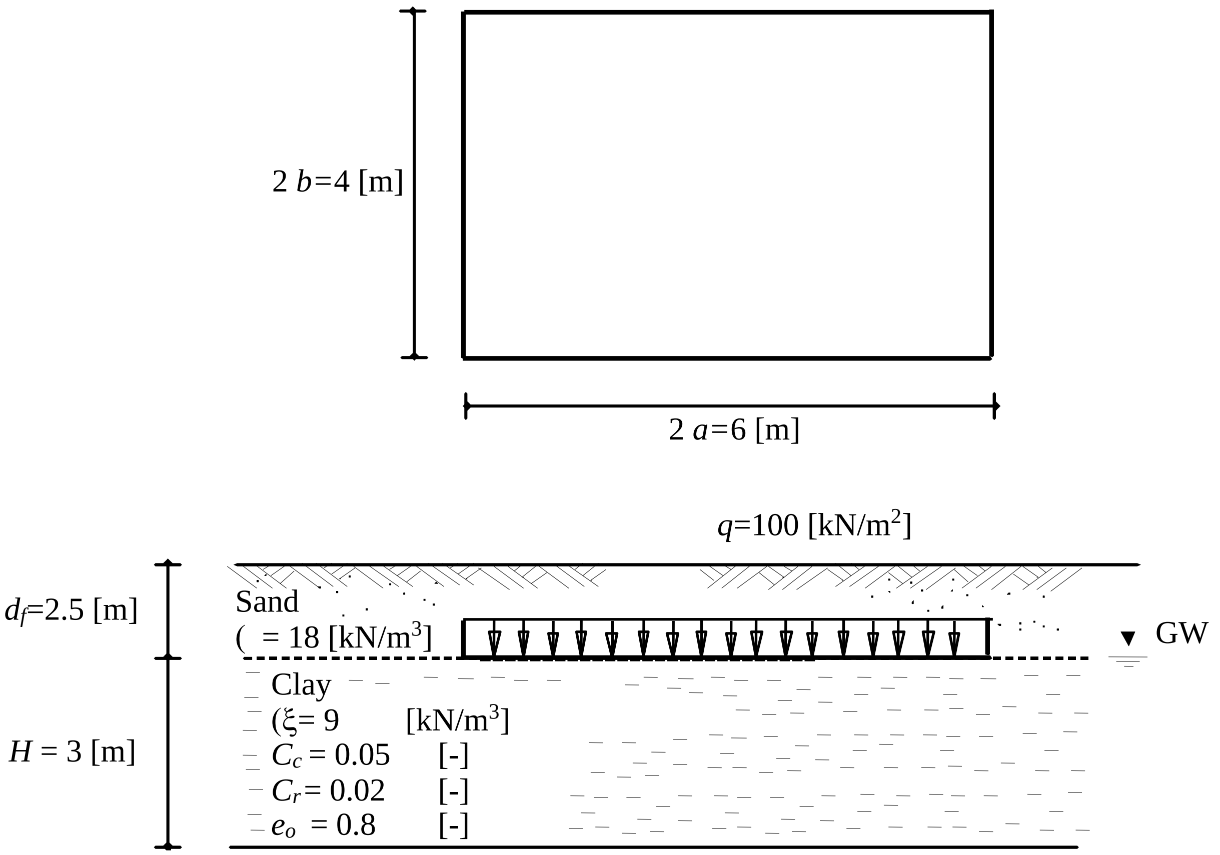


Figure 4.32 Raft with dimensions and soil profile

Table 4.17 Soil data

Unit weight of the soil above the water level	γ	= 18	[kN/m^3]
Unit weight of the soil under the water level	$\gamma\xi$	= 9	[kN/m^3]
Recompression index	C_r	= 0.02	[-]
Compression index	C_c	= 0.05	[-]
Initial void ratio	e_o	= 0.8	[-]

4.10.9.2 Hand calculation

4.10.9.2.1 Calculation with Loading and reloading pressure

The reloading pressure due to the soil excavation is $q_v = \gamma .\ d_f = 18 \times 2.5 = 45$ [kN/m^2].
The total applied pressure on the soil due to the raft is $q = 100$ [kN/m^2].

4.10.9.2.2 Overburden pressure

The overburden pressure on the top surface of the clay layer is given by:

$$\sigma_o = \gamma d_f = 18 \times 2.5 = 45\,[\text{kN/m}^2]$$

The overburden pressure in the middle of the layer is given by:

$$\sigma_o = \gamma d_f + \frac{\gamma' H}{2} = 18 \times 2.5 + \frac{9 \times 3}{2} = 58.5\,[\text{kN/m}^2]$$

4.10.9.2.3 Increment of vertical stress

For a rectangular loaded area of intensity q [kN/m^2] with sides a [m] and b [m] acting on the surface, the increment of vertical stress $\Delta\sigma$ [kN/m^2] in the soil layer of thickness h under the corner of the rectangular can be expressed as:

$$\Delta\sigma = \frac{q}{2\pi h}\left\{\left(b \ln \frac{(c_2 - a)(m + a)}{(c_2 + a)(m - a)} + a \ln \frac{(c_2 - b)(m + b)}{(c_2 + b)(m - b)} + h_2 \tan^{-1}\frac{ab}{h_2 c_2}\right) - \left(b \ln \frac{(c_1 - a)(m + a)}{(c_1 + a)(m - a)} + a \ln \frac{(c_1 - b)(m + b)}{(c_1 + b)(m - b)} + h_1 \tan^{-1}\frac{ab}{h_1 c_1}\right)\right\}$$

where: $m = \sqrt{a^2 + b^2}$, $c_2 = \sqrt{a^2 + b^2 + h_2^2}$ and $c_1 = \sqrt{a^2 + b^2 + h_1^2}$

h Thickness of the soil layer, [m].
h_1 Depth of the layer top from the surface, [m].
h_2 Depth of the layer bottom from the surface, [m].

when $h_1 = 0$, the above equation becomes:

$$\Delta\sigma = \frac{q}{2\pi h}\left\{\left(a \ln \frac{(c_2 - b)(m + b)}{(c_2 + b)(m - b)} + b \ln \frac{(c_2 - a)(m + a)}{(c_2 + a)(m - a)} + h_2 \tan^{-1}\frac{ab}{h_2 c_2}\right)\right\}$$

and

$$m = \sqrt{a^2 + b^2} = \sqrt{3^2 + 2^2} = 3.606$$
$$c_2 = \sqrt{a^2 + b^2 + h_2^2} = \sqrt{3^2 + 2^2 + 3^2} = 4.69$$

The stress under the corner of the quarter of the raft is:

$$\Delta\sigma = \frac{q}{2\pi 3}\left\{\left(3 \ln \frac{(4.69 - 2)(3.606 + 2)}{(4.69 + 2)(3.606 - 2)} + 3 \ln \frac{(4.69 - 3)(3.606 + 3)}{(4.69 + 3)(3.606 - 3)} + 3 \tan^{-1}\frac{3 \times 2}{3 \times 4.69}\right)\right\} = I_\sigma q$$

$$\Delta\sigma = 0.2108q$$

$\therefore$ The increment of vertical stress in the entire soil layer due to the pressure q is given by:

$$\Delta\sigma_{av1} = 4 I_\sigma q = 4 \times 0.2108 \times q = 0.8432\,q$$

$\therefore$ The increment of vertical stress in the soil layer due to the reloading pressure q_v is given by:

$$\Delta\sigma_{av1} = 0.8432\,q_v = 0.8432 \times 45 = 37.94\,[\text{kN/m}^2]$$

∴ Increment of vertical stress in the soil layer due to the total applied pressure q is given by:

$$\Delta\sigma_{av} = 0.8432\, q_o = 0.8432 \times 100 = 84.32\ [\text{kN/m}^2]$$

4.10.9.2.4 Preconsolidation pressure of the layer σc

Preconsolidation pressure of the layer σ_c is given by:

$$\sigma_c = OCR \times \sigma_o$$
$$\sigma_c = 1.65 \times 58.5 = 96.44\ [\text{kN/m}^2]$$

4.10.9.2.5 Consolidation settlement:

The total settlement of the raft occurred due to two parts. Reloading part due to the reloading pressure of q_v = 45 [kN/m^2], while loading part due to the pressure from q_v = 45 [kN/m^2] to the total pressure q = 100 [kN/m^2]. This settlement can be obtained from the following equation:

$$Sc = \frac{C_r\, h}{1 + e_o} \log\left(\frac{\sigma_c}{\sigma_o}\right) + \frac{C_c\, h}{1 + e_o} \log\left(\frac{\sigma_o + \Delta\sigma_{va}}{\sigma_c}\right)$$

where:

Sc	Consolidation settlement, [m].
h	Layer thickness, [m].
C_r	Recompression index, [-].
C_c	Compression index, [-].
e_o	Initial void ratio, [-].
σ_o	Overburden pressure in the middle of the layer, [kN/m^2].
$\Delta\sigma_{va}$	Increment of vertical stress in the soil layer due to the total applied pressure q, [kN/m^2].
$\Delta\sigma_{va1}$	Increment of vertical stress in the soil layer due to the reloading pressure q_v, [kN/m^2].

The consolidation settlement is given by:

$$S_c = \frac{0.02 \times 3}{1 + 0.8} \log\left(\frac{96.44}{58.5}\right) + \frac{0.05 \times 3}{1 + 0.8} \log\left(\frac{58.5 + 84.32}{96.44}\right)$$

$$S_c = 0.0072 + 0.0142\ [\text{m}]$$

$$S_c = 0.72 + 1.42 = 2.14\ [\text{cm}]$$

where:

Reloading settlement is	S_u = 0.72 [cm]
Loading settlement is	S_e = 1.42 [cm]
Total consolidation settlement is	S_c = 2.14 [cm]

4.10.9.2.6 Calculation with recompression index only (C_c=C_r=0.02 [-])

The settlement of the raft is obtained from the compression index only, in which the compression index is equal to the recompression index, C_c=C_r=0.02 [-]).

4.10.9.2.7 Preconsolidation pressure of the layer σ_c

Preconsolidation pressure of the layer σ_c is given by:

$$\sigma_c = OCR \times \sigma_o$$
$$\sigma_c = 2.5 \times 58.5 = 146.25\,[\text{kN/m}^2] > 142.82\,[\text{kN/m}^2]$$
$$\sigma_c > \sigma_o + \Delta\sigma_{va}$$

This settlement can be obtained from the following equation $\sigma_f < \sigma_c$:

$$Sc = \frac{C_r\, h}{1+e_o} \log\left(\frac{\sigma_o + \Delta\sigma_{va}}{\sigma_o} \right)$$

The consolidation settlement is given by:

$$S_c = \frac{0.02 \times 3}{1+0.8} \log\left(\frac{58.5 + 84.32}{58.5} \right)$$

$$S_c = 0.0129\ [\text{m}] = 1.29\ [\text{cm}]$$

4.10.9.2.8 Calculation with compression index only (C_r=C_c=0.05 [-])

The settlement of the raft is obtained from the recompression index only, in which the recompression index is equal to the compression index, C_r=C_c=0.05 [-]).

4.10.9.2.9 Preconsolidation pressure of the layer σ_c

Preconsolidation pressure of the layer σ_c is given by:

$$\sigma_c = OCR \times \sigma_o$$
$$\sigma_c = 1.0 \times 58.5 = 58.5\,[\text{kN/m}^2] < 142.82\,[\text{kN/m}^2]$$
$$\sigma_c < \sigma_o + \Delta\sigma_{va}$$

This settlement can be obtained from the following equation $\sigma_f > \sigma_c$:

$$Sc = \frac{C_c\, h}{1+e_o} \log\left(\frac{\sigma_o + \Delta\sigma_{va}}{\sigma_o} \right)$$

The consolidation settlement is given by:

$$S_c = \frac{0.05 \times 3}{1 + 0.8} \log\left(\frac{58.5 + 84.32}{58.5} \right)$$

$$S_c = 0.0323 \text{ [m]} = 3.23 \text{ [cm]}$$

4.10.9.3 Consolidation settlement by GEO Tools

The consolidation settlements obtained from *GEO Tools* at the center of the raft are 2.14 [cm], 1.29 [cm] and 3.23 [cm] for the three cases, respectively. They are the same as those of the hand calculations. The input data and results of *GEO Tools* are presented on the next pages.

```
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Loading and reloading settlement under a rectangular raft
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex8

------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
------------------------------------------------------------

Data:
Rectangular load                      q        [kN/m2] = 100.0000
Length                                a        [m]     = 6.00
Width                                 b        [m]     = 4.00

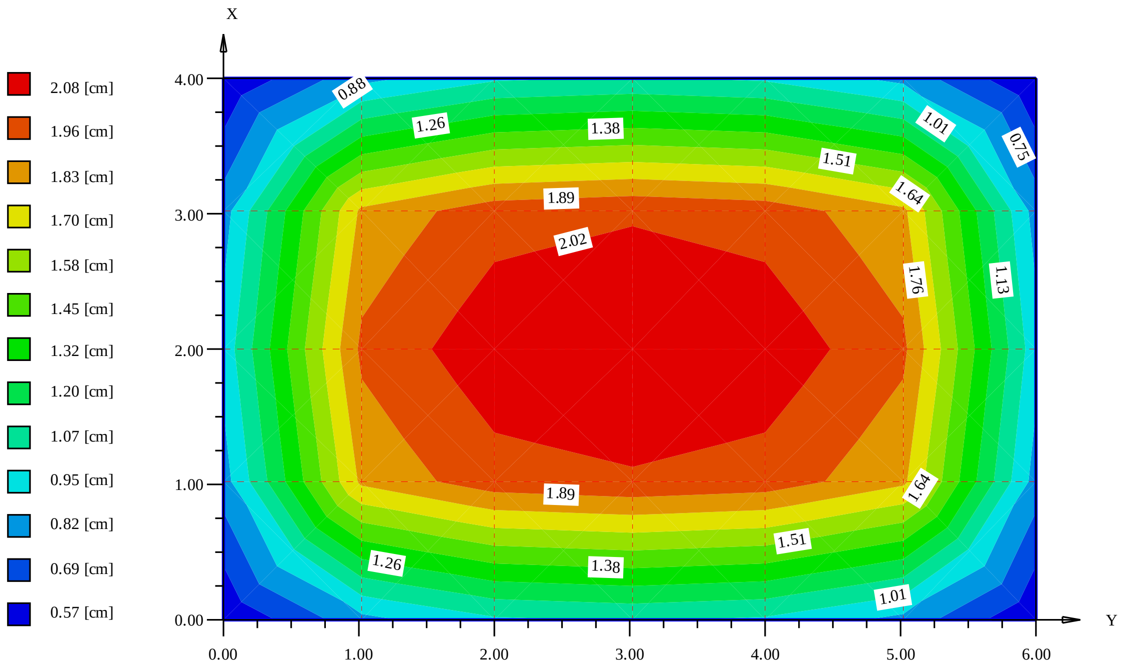
Soil Data:
Layer No.: 1
Compression index                     Cc       [-]     = 0.050
Recompression Index                   Cr       [-]     = 0.020
Over consolidated ratio               OCR      [-]     = 1.650
Initial void ratio                    eo       [-]     = 0.800
Unit weight                           Gamma_c  [kN/m3] = 9.00000
Layer thickness                       h        [m]     = 3.00


Overburden pressure                   Gamma*z  [kN/m2] = 45.0000
Depth increment in z-direction        Dz       [m]     = 0.50

Point coordinates:
X-coord.                              x        [m]     = 3.00
Y-coord.                              y        [m]     = 2.00
Y-coord.                              z        [m]     = 0.00

Results:
Settlement
Layer No.: 1                          S1       [cm]    = 2.14
Total                                 St       [cm]    = 2.14
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 2.14 at node 18, Min. s = 0.50 at node 1

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Scale: 41	**Project: User's Manual**
File: 4Con_Ex8	**Date: 01-09-2017**
Page No.:	**Title: Loading and reloading settlement under a rectangular raft**

```
************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Loading and reloading settlement under a rectangular raft
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex8

------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
------------------------------------------------------------

Data:
Rectangular load                      q       [kN/m2] = 100.0000
Length                                a       [m]     = 6.00
Width                                 b       [m]     = 4.00

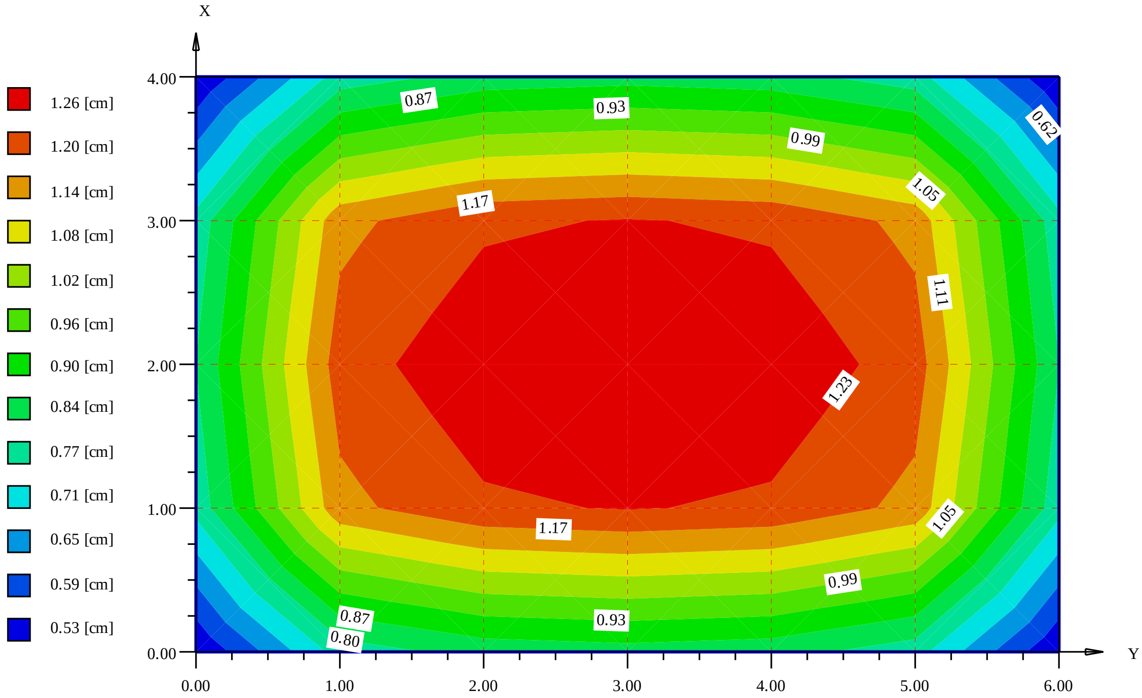
Soil Data:
Layer No.: 1
Compression index                     Cc      [-]     = 0.020
Recompression Index                   Cr      [-]     = 0.020
Over consolidated ratio               OCR     [-]     = 2.500
Initial void ratio                    eo      [-]     = 0.800
Unit weight                           Gamma_c [kN/m3] = 9.00000
Layer thickness                       h       [m]     = 3.00


Overburden pressure                   Gamma*z [kN/m2] = 45.0000
Depth increment in z-direction        Dz      [m]     = 0.50

Point coordinates:
X-coord.                              x       [m]     = 3.00
Y-coord.                              y       [m]     = 2.00
Y-coord.                              z       [m]     = 0.00

Results:
Settlement
Layer No.: 1                          S1      [cm]    = 1.29
Total                                 St      [cm]    = 1.29
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 1.29 at node 18, Min. s = 0.50 at node 1

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Scale: 38	**Project: User's Manual**
File: 4Con_Ex8	**Date: 01-09-2017**
Page No.:	**Title: Loading and reloading settlement under a rectangular raft**

```
*************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
*************************************************************
Title: Loading and reloading settlement under a rectangular raft
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex8

-------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
-------------------------------------------------------------

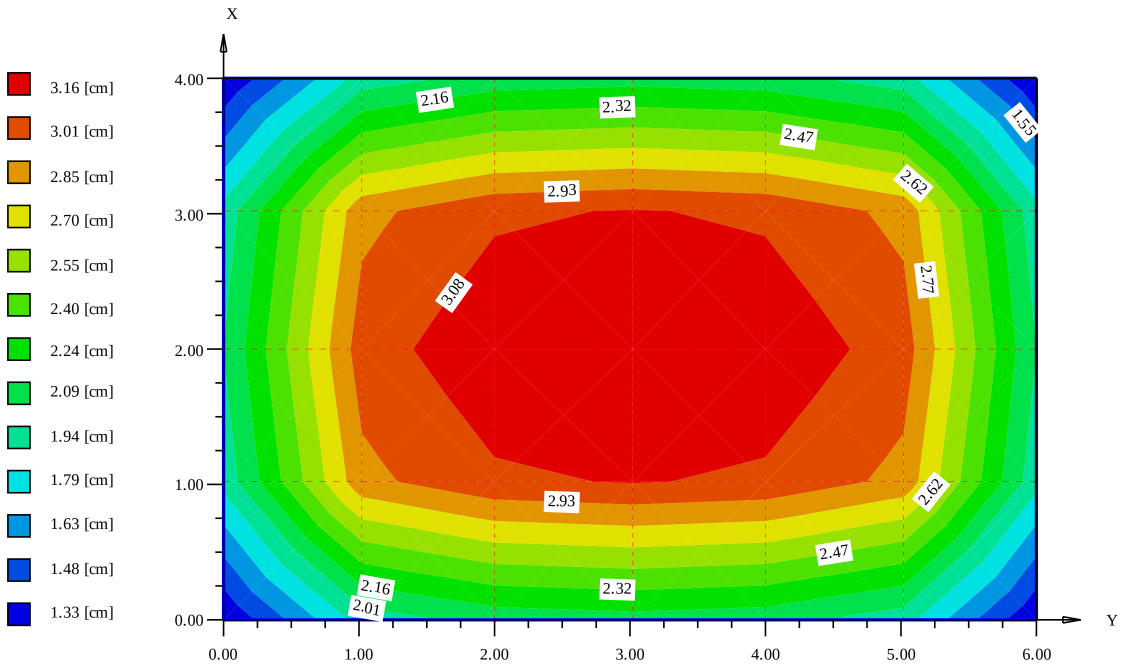
Data:
Rectangular load                      q       [kN/m2] = 100.0000
Length                                a       [m]     = 6.00
Width                                 b       [m]     = 4.00

Soil Data:
Layer No.: 1
Compression index                     Cc      [-]     = 0.050
Recompression Index                   Cr      [-]     = 0.050
Over consolidated ratio               OCR     [-]     = 1.000
Initial void ratio                    eo      [-]     = 0.800
Unit weight                           Gamma_c [kN/m3] = 9.00000
Layer thickness                       h       [m]     = 3.00


Overburden pressure                   Gamma*z [kN/m2] = 45.0000
Depth increment in z-direction        Dz      [m]     = 0.50

Point coordinates:
X-coord.                              x       [m]     = 3.00
Y-coord.                              y       [m]     = 2.00
Y-coord.                              z       [m]     = 0.00

Results:
Settlement
Layer No.: 1                          S1      [cm]    = 3.23
Total                                 St      [cm]    = 3.23
```



Settlements s [cm]

Max. s = 3.23 at node 18, Min. s = 1.25 at node 1

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Scale: 41	**Project: User's Manual**
File: 4Con_Ex8	**Date: 01-09-2017**
Page No.:	**Title: Loading and reloading settlement under a rectangular raft**

4.10.10Example 9: Consolidation settlement under a rectangular footing

4.10.10.1 Description of the problem

To verify the consolidation settlement calculated by *GEO Tools*, the final consolidation settlement of a clay layer under a rectangular footing calculated by *Das* (1999), Example 5.6, page 236, is compared with that obtained by *GEO Tools*.

A rectangular footing 1.5 [m] × 3.0 [m] in plan at a depth of 1.0 [m] below the ground surface is considered as shown in Figure 4.33. Water table is located at 2.5 [m] below the ground surface. The contact pressure under the footing is assumed to be uniformly distributed and equal to q = 170 [kN/m^2]. A normally consolidated clay layer 3 [m] thick is located at a depth of 4.0 [m] below the ground surface. The soil profile is shown in Figure 4.33, while the soil properties are shown in Table 4.18. It is required to determine the final settlement under the center of the footing due to consolidation of the clay.

Table 4.18 Soil properties

Layer No.	Type of Soil	Depth of the layer under the ground surface z [m]	Unit weight of the soil γ [kN/m^3]	Compression index C_c [-]	Void ratio e_o [-]
1	Sand	2.5	16.5	-	-
2	Sand	4.0	17.8	-	-
3	Clay	7.0	18.2	0.27	0.92

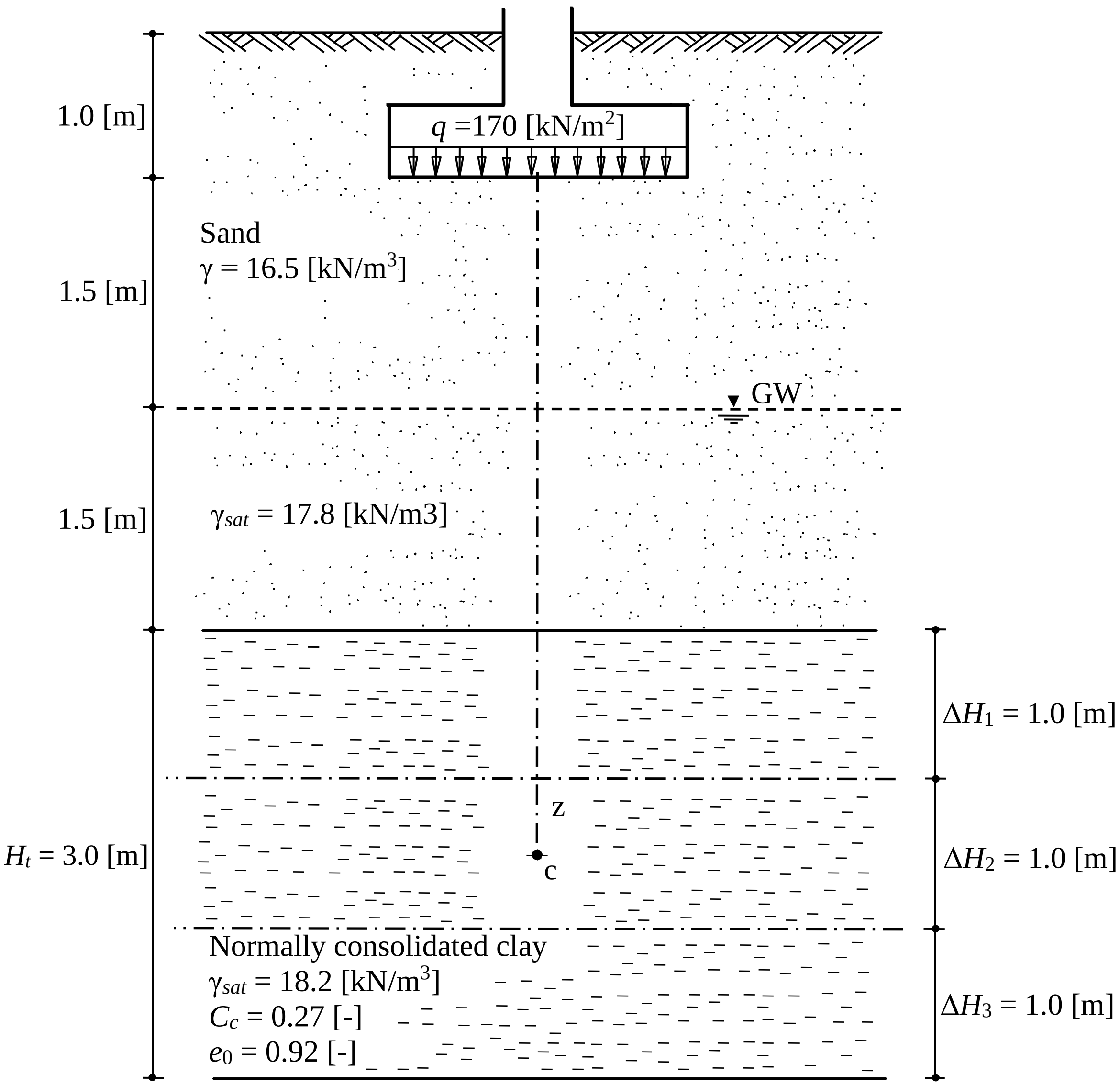


Figure 4.33 Soil profile

4.10.10.2 Hand calculation of consolidation

According to *Das* (1999), the consolidation of the clay layer can be obtained by the hand calculation as follows:

The clay layer is thick relative to the dimensions of the footing. Therefore, the clay layer is divided into three layers each 1.0 [m] thick.

4.10.10.2.1 Calculation of the effective stress $\sigma'_{o(i)}$

The effective stress σ'_o on the top of the clay layer is

$$\sigma'_o = \gamma_1 \, z_1 + \gamma_2 \, z_2$$

$$\sigma'_o = 16.5 \times 2.5 + (17.8 - 9.81) \times 1.5 = 53.24 \, [\text{kN/m}^2]$$

The effective stress $\sigma'_{o(i)}$ at the middle of each sub layer is calculated in Table 4.19.

Table 4.19 Calculation of effective stress $\sigma'_{o(i)}$

Layer No.	Layer thickness ΔH_i [m]	Depth of the middle of the clay sub layer z [m]	Unit weight of the soil $\sigma'_{o(i)}$ [kN/m^2]
1	1	1+1.5+1.5+0.5=4.5	(1+1.5)16.5+1.5(17.8-9.81)+0.5(18.2-9.81) =57.43
2	1	4.5+1=5.5	57.43+1(18.2-9.81)= 65.82
3	1	5.5+1=6.5	65.82+1(18.2-9.81)= 74.21

4.10.10.2.2 Calculation of the increase of effective stress $\Delta\sigma'_i$

For a rectangular loaded area of dimension B=1.5 [m], L=3.0 [m] and q = 170 [kN/m^2], the increase of effective stress $\Delta\sigma'_i$ below the center of the footing at the middle of each sub-layer is calculated in Table 4.19Table 4.20 according to Table 5.3 in *Das* (1999).

Table 4.20 Steps of calculation of increase of effective stress $\Delta\sigma'_i$

Layer No.	Layer thickness ΔH_i [m]	Depth to the middle of the clay sub layer from bottom of the foundation z [m]	L/B [-]	z/B [-]	Increase of effective stress $\Delta\sigma'_i$ [kN/m^2
1	1.0	3.5	2	2.33	27.2
2	1.0	4.5	2	3.0	16.15
3	1.0	5.5	2	3.67	11.9

4.10.10.2.3 Calculation of consolidation settlement s_c

The consolidation settlement S_c is given by:

$$S_c = \frac{C_c H_i}{1+e_o} \sum_{i=1}^{3} \log\left(\frac{\sigma'_{o(i)} + \Delta\sigma'_{o(i)}}{\sigma'_{o(i)}}\right)$$

$$S_c = \frac{0.27 \times 1}{1 + 0.9}\left[\log\left(\frac{57.43 + 27.2}{57.43}\right) + \log\left(\frac{65.82 + 16.15}{65.82}\right) + \log\left(\frac{74.21 + 11.9}{74.21}\right)\right]$$

$$S_c = 0.142[0.168 + 0.096 + 0.065]$$

The total consolidation settlement obtained by the hand calculation is:

$$s_c = 0.047 \text{ [m]} = 4.7 \text{ [cm]}$$

4.10.10.3 Consolidation by GEO Tools

The final consolidation settlement of the clay under the center of the footing obtained by the program *GEO Tools* is s_c = 4.4 [cm] and nearly equal to that obtained by the hand calculation. The input data and results of *GEO Tools* are presented on the next pages.

```
*************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
*************************************************************
Title: Consolidation settlement under a rectangular footing
Date: 01-09-2017
Project: Das (1999), Example 5.6, page 236
File: 4Con_Ex9

-------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
-------------------------------------------------------------

Data:
Rectangular load                       q       [kN/m2] = 170.00
Length                                 a       [m]     = 3.00
Width                                  b       [m]     = 1.50

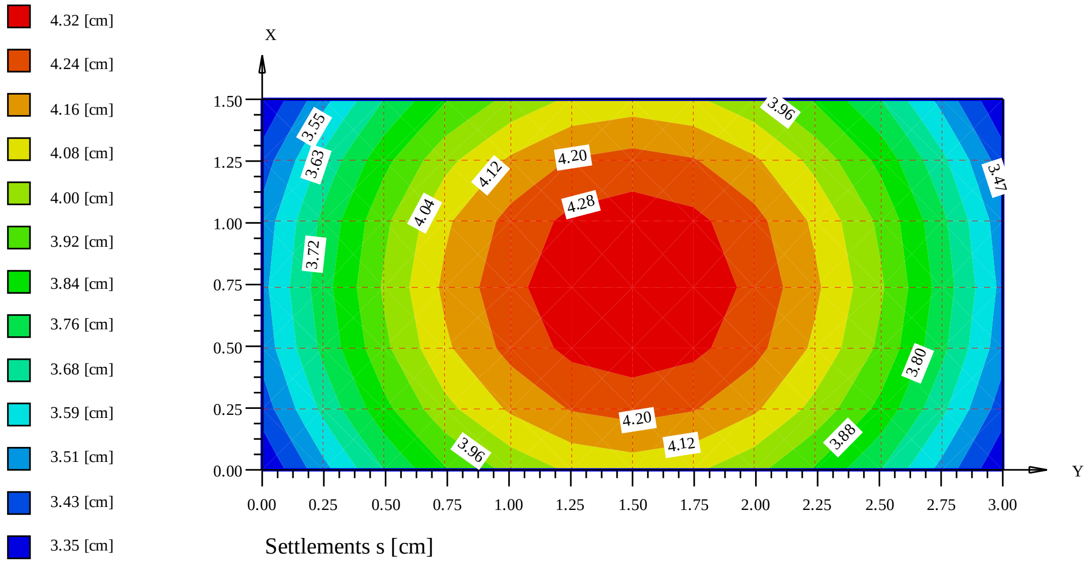
Soil Data:
Layer No.: 1
Compression index                      Cc      [-]     = 0.27
Recompression Index                    Cr      [-]     = 0.27
Over consolidated ratio                OCR     [-]     = 1.00
Initial void ratio                     eo      [-]     = 0.92
Unit weight                            Gamma_c [kN/m3] = 8.39
Layer thickness                        h       [m]     = 3.00


Overburden pressure                    Gamma*z [kN/m2] = 53.24
Depth increment in z-direction         Dz      [m]     = 0.00

Point coordinates:
X-coord.                               x       [m]     = 1.50
Y-coord.                               y       [m]     = 0.75
Y-coord.                               z       [m]     = 3.00

Results:
Settlement
Layer No.: 1                           S1      [cm]    = 4.36
Total                                  St      [cm]    = 4.36
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 4.36 at node 46, Min. s = 3.31 at node 1

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Scale: 22	**Project: Das (1999), Example 5.6, page 236**
File: 4Con_Ex9	**Date: 01-09-2017**
Page No.:	**Title: Consolidation settlement under a rectangular footing**

4.10.11 Example 10: Consolidation settlement of normally consolidated clay

4.10.11.1 Description of the problem

To verify the consolidation settlement of normally consolidated clay calculated by program *GEO Tools*, a hand calculation of a consolidation settlement for a clay layer under a square footing is compared with that obtained by *ELPLA*.

A square footing of B = 6 [ft] side located at a depth of d_f =5 [ft] below the ground surface is considered as shown in Figure 4.34. Ground water table lies at a depth z = 10 [ft] under the ground surface. The contact pressure under the footing is assumed to be uniformly distributed and equal to q = 1.39 [kip/ft^2]. Soil profile is consists of a sand layer of thickness h_s =10 [ft] above a normally consolidated clay layer of thickness h_c = 8 [ft]. Figure 4.34 shows the footing with dimensions and soil profile, while Table 4.21 shows the soil data. It is required to determine the final settlement under the center of the footing due to consolidation of the clay.

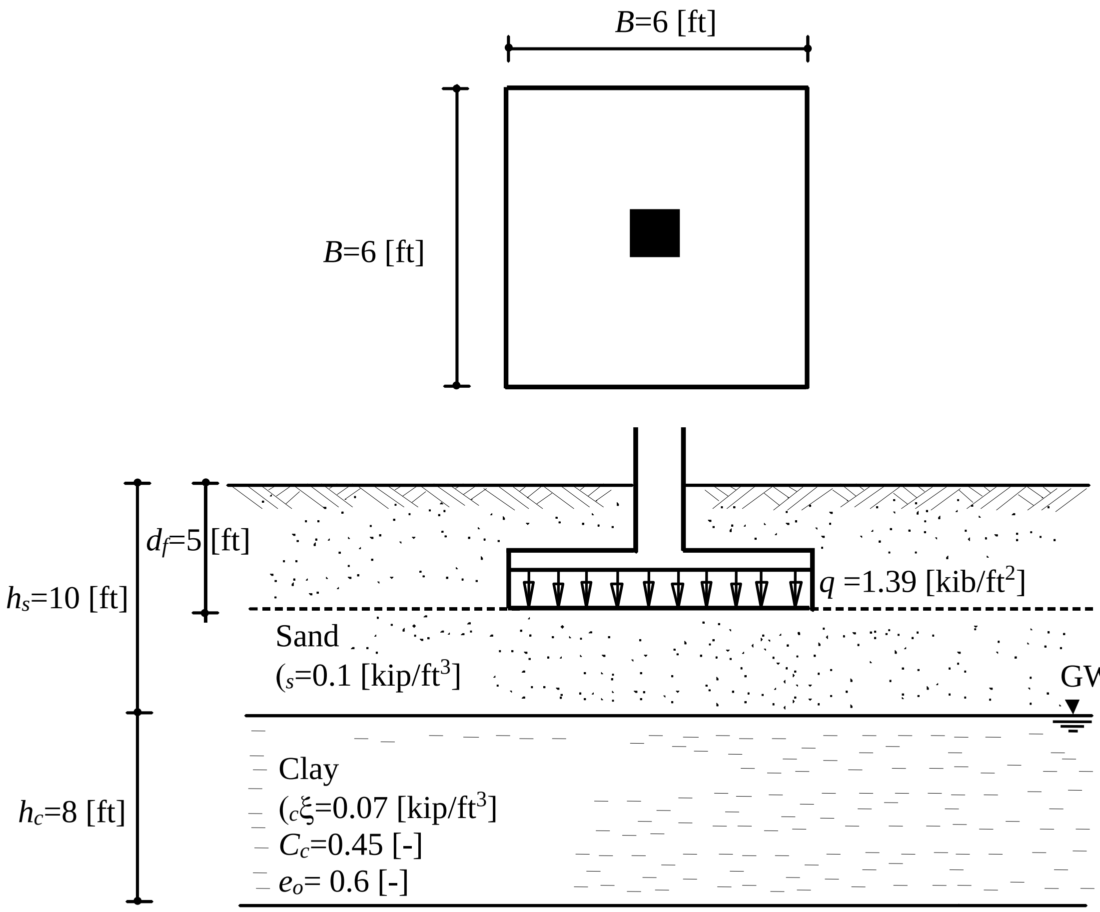


Figure 4.34 Footing with dimensions and soil profile

Table 4.21 Soil data

Unit weight of the sand above the water level	γ_s	= 0.1 [kip/ft^3]
Unit weight of the clay under the water level	$\gamma_c\xi$	= 0.07 [kip/ft^3]
Compression index	C_c	= 0.45 [-]
Initial void ratio	e_o	= 0.6 [-]

4.10.11.2 Hand calculation

4.10.11.2.1 Overburden pressure

The overburden pressure on the top surface of the clay layer is given by:

$$\sigma_o = \gamma_s h_s = 0.1 \times 10 = 1 \text{ [kip/ft}^2\text{]}$$

The overburden pressure in the middle of the clay layer is given by:

$$\sigma_o = \gamma_s h_s + \frac{\gamma_c' h_c}{2} = 0.1 \times 10 + \frac{0.07 \times 8}{2} = 1.28 \text{ [kip/ft}^2\text{]}$$

4.10.11.2.2 Increment of vertical stress

The average stress in the clay layer is calculated numerically using Simpson's rule from the vertical stresses at three levels in the clay layer.

Stress σ_z at a depth z in the soil under the center of a rectangular loaded area of sides $2a \times 2b$ is given by:

$$\sigma_z = \frac{4q}{2\pi}\left[\left(\frac{1}{R_1^2} + \frac{1}{R_2^2}\right)\frac{a.b.z}{R_3} + \tan^{-1}\frac{a.b}{z.R_3}\right]$$

where:

$$R_1^2 = \sqrt{a^2 + z^2}$$

$$R_2^2 = \sqrt{b^2 + z^2}$$

$$R_3 = \sqrt{a^2 + b^2 + z^2}$$

Stress σ_z at the surface of the clay layer:

Depth from bottom of footing to surface of clay layer = 10-5= 5 [ft]

$$R_1^2 = R_2^2 = 3^2 + 5^2 = 34$$

$$R_3 = \sqrt{3^2 + 3^2 + 5^2} = 6.56$$

$$\sigma_z = \frac{4 \times 1.39}{2\pi}\left[\left(\frac{1}{34} + \frac{1}{34}\right)\frac{3 \times 3 \times 5}{6.56} + \tan^{-1}\frac{3 \times 3}{5 \times 6.56}\right] = 0.594 \text{ [kip/ft}^2\text{]}$$

Stress σ_z at the middle of the clay layer:

Depth from bottom of footing to middle of clay layer = 10-5+8/2= 9 [ft]

$$R_1^2 = R_2^2 = 3^2 + 9^2 = 90$$
$$R_3 = \sqrt{3^2 + 3^2 + 9^2} = 9.95$$
$$\sigma_z = \frac{4 \times 1.39}{2\pi}\left[\left(\frac{1}{90} + \frac{1}{90}\right)\frac{3 \times 3 \times 9}{9.95} + \tan^{-1}\frac{3 \times 3}{9 \times 9.95}\right] = 0.249 \text{ [kip/ft}^2\text{]}$$

Stress σ_z at the bottom of the clay layer:

Depth from bottom of footing to bottom of clay layer = 10-5+8= 13 [ft]

$$R_1^2 = R_2^2 = 3^2 + 13^2 = 178$$
$$R_3 = \sqrt{3^2 + 3^2 + 13^2} = 13.67$$
$$\sigma_z = \frac{4 \times 1.39}{2\pi}\left[\left(\frac{1}{178} + \frac{1}{178}\right)\frac{3 \times 3 \times 13}{13.67} + \tan^{-1}\frac{3 \times 3}{13 \times 13.67}\right] = 0.130 \text{ [kip/ft}^2\text{]}$$

Average stress $\Delta\sigma_z$ in the clay layer by numerical integration using Simpson's rule:

$$\Delta\sigma_z = \frac{h_c}{2}\left[\frac{\sigma_z(Top) + \sigma_z(bottom)}{2} + \sigma_z(midell)\right]$$
$$\Delta\sigma_z = \frac{1}{2}\left[\frac{0.594 + 0.130}{2} + 0.249\right] = 0.306 \text{ [kip/ft}^2\text{]}$$

4.10.11.2.3 Consolidation settlement:

The consolidation settlement of normally consolidated clay layer is obtained from:

$$Sc = \frac{C_c\, h}{1 + e_o} \log\left(\frac{\sigma_o + \Delta\sigma_{va1}}{\sigma_o}\right)$$

where:

Sc Consolidation settlement, [ft].
h Layer thickness, [ft].
C_c Compression index, [-].
e_o Initial void ratio, [-].
σ_o Overburden pressure in the middle of the layer, [kib/ft^2].
$\Delta\sigma_{va}$ Increment of vertical stress in the soil layer due to the applied pressure q, [kib/ft^2].

The consolidation settlement is given by:

$$S_c = \frac{0.45 \times 8}{1 + 0.6} \log\left(\frac{1.28 + 0.306}{1.28} \right)$$

$$S_c = 0.209 \text{ [ft]} = 0.209 \times 12 = 2.51 \text{[in]}$$

4.10.11.3 Consolidation settlement by ELPLA

The exact consolidation settlement obtained from *GEO Tools* at the center of the footing is 2.42 [in] nearly the same as that of the hand calculation with a difference of 0.09 [in]. The input data and results of *GEO Tools* are presented on the next pages.

```
************************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Consolidation settlement of normally consolidated clay
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex10

------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
------------------------------------------------------------

Data:
Rectangular load                      q       [kip/ft2] = 1.39
Length                                a       [ft]      = 6.00
Width                                 b       [ft]      = 6.00

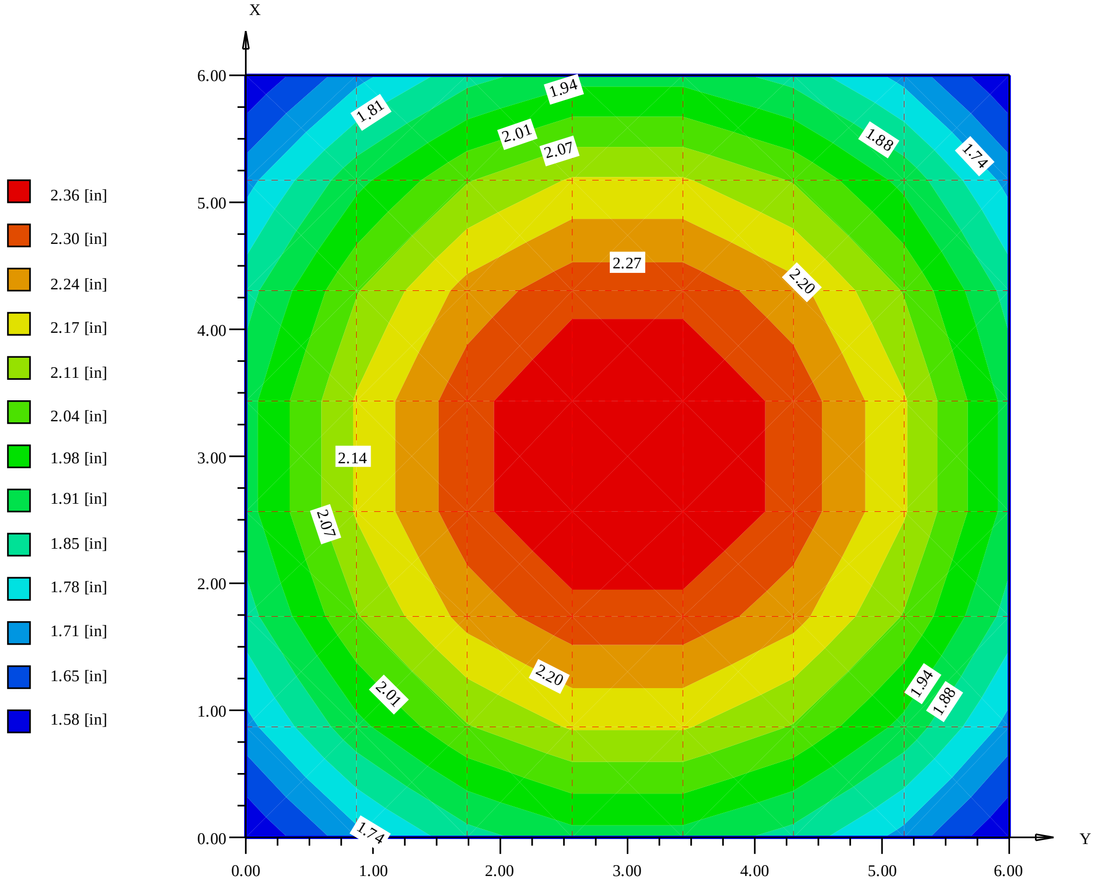
Soil Data:
Layer No.: 1
Compression index                     Cc      [-]       = 0.45
Recompression Index                   Cr      [-]       = 0.45
Over consolidated ratio               OCR     [-]       = 1.00
Initial void ratio                    eo      [-]       = 0.60
Unit weight                           Gamma_c [kip/ft3] = 0.07
Layer thickness                       h       [ft]      = 8.00


Overburden pressure                   Gamma*z [kip/ft2] = 1.00
Depth increment in z-direction        Dz      [ft]      = 0.00

Point coordinates:
X-coord.                              x       [ft]      = 3.00
Y-coord.                              y       [ft]      = 3.00
Y-coord.                              z       [ft]      = 5.00

Results:
Settlement
Layer No.: 1                          S1      [in]      = 2.42
Total                                 St      [in]      = 2.42
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [in]
Max. s = 2.40 at node 37, Min. s = 1.55 at node 1

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Scale: 13	**Project: User's Manual**
File: 4Con_Ex10	**Date: 01-09-2017**
Page No.:	**Title: Consolidation settlement of normally consolidated clay**

4.10.12Example 11: Consolidation settlement of overly consolidated clay

4.10.12.1 Description of the problem

To verify the consolidation settlement of overly consolidated clay calculated by program *GEO Tools*, a hand calculation of a consolidation settlements for a clay layer under a square footing are compared with those obtained by *ELPLA*.

A square footing of B = 6 [ft] side located at a depth of d_f =3 [ft] below the ground surface is considered as shown in Figure 4.35. Ground water table lies at a depth z = 8 [ft] under the ground surface. The contact pressure under the footing is assumed to be uniformly distributed. Soil profile is consists of a sand layer of thickness h_s =8 [ft] above an overly consolidated clay layer of thickness h_c = 8 [ft]. Figure 4.35 shows the footing with dimensions and soil profile, while Table 4.22 shows the soil data.

It is required to determine the final settlement under the center of the footing due to consolidation of the clay for the following two cases of loading:

a) Consolidation settlement of overly consolidated clay (σ_f<σ_c)
Column load P= 40 [kib]
Considering a compression index only, C_c=C_r=0.1 [-].

b) Consolidation settlement of overly consolidated clay (σ_f >σ_c>σ_o)
Column load P= 80 [kib]
Considering compression and recompression indices, C_r=0.1 [-], C_c=0.5 [-].

Table 4.22 Soil data

Unit weight of the sand above the water level	γ_s	= 0.12	[kip/ft^3]
Unit weight of the clay under the water level	$\gamma_c\xi$	= 0.08	[kip/ft^3]
Recompression index	C_r	= 0.1	[-]
Compression index	C_c	= 0.5	[-]
Initial void ratio	e_o	= 0.7	[-]
Preconsolidation pressure	σ_c	=1.6	[kip/ft^2]

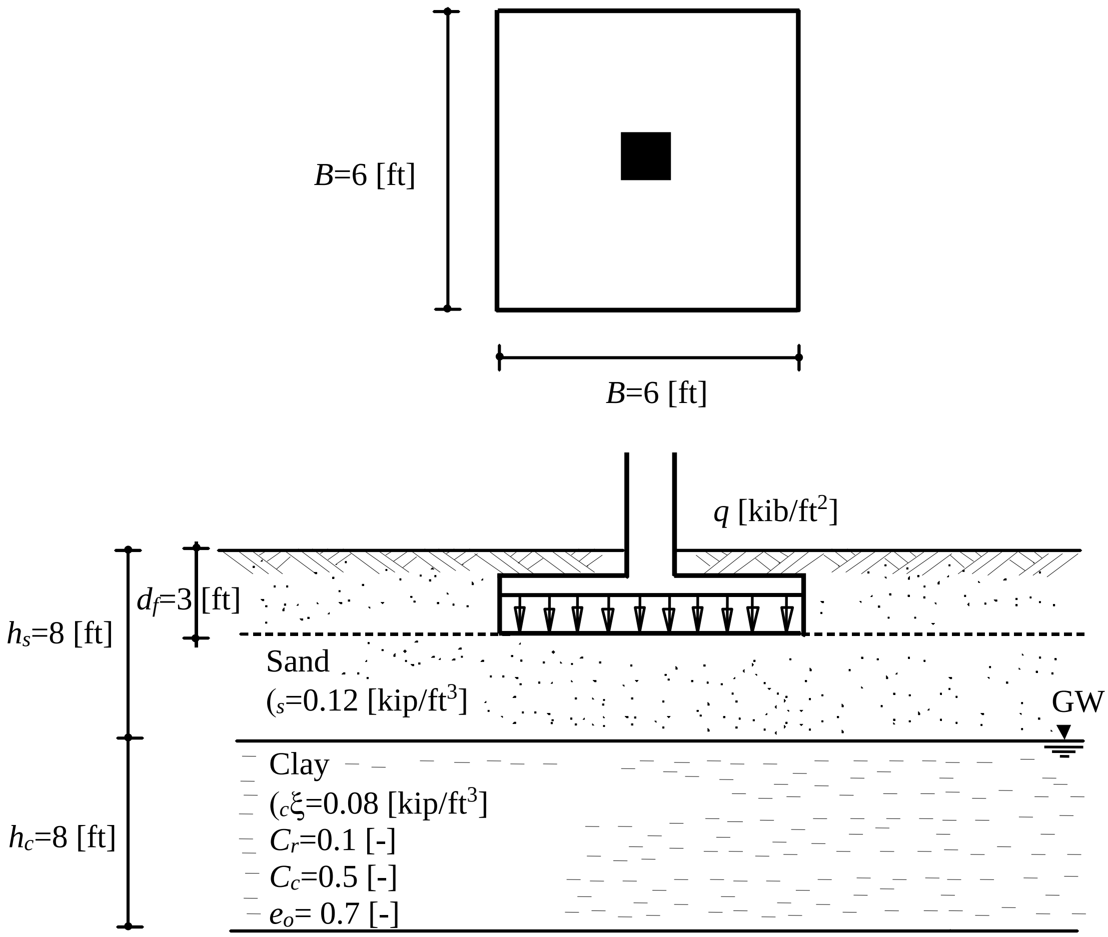


Figure 4.35 Footing with dimensions and soil profile

4.10.12.2 Hand calculation

4.10.12.2.1 Overburden pressure

The overburden pressure on the top surface of the clay layer is given by:

$$\sigma_o = \gamma_s h_s = 0.12 \times 8 = 0.96 \ \text{[kip/ft}^2\text{]}$$

The overburden pressure in the middle of the clay layer is given by:

$$\sigma_{o1} = \sigma_o + \frac{\gamma_c' h_c}{2} = 0.96 + \frac{0.08 \times 8}{2} = 1.28 \ \text{[kip/ft}^2\text{]}$$

4.10.12.2.2 Increment of vertical stress

The average stress in the clay layer is calculated numerically using Simpson's rule from the vertical stresses at three levels in the clay layer.

Stress σ_z at a depth z in the soil under the center of a rectangular loaded area of sides $2a \times 2b$ is given by:

$$\sigma_z = \frac{4}{2\pi}\frac{P}{2a \times 2b}\left[\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}\right)\frac{a.b.z}{R_3}+\tan^{-1}\frac{a.b}{z.R_3}\right]$$

where:

$$R_1^2 = \sqrt{a^2+z^2}$$
$$R_2^2 = \sqrt{b^2+z^2}$$
$$R_3 = \sqrt{a^2+b^2+z^2}$$

Stress σ_z at the surface of the clay layer:

Depth from bottom of footing to surface of clay layer = 8-3= 5 [ft]

$$R_1^2 = R_2^2 = 3^2+5^2 = 34$$
$$R_3 = \sqrt{3^2+3^2+5^2} = 6.56$$
$$\sigma_z = \frac{4\times P}{2\pi\times 6\times 6}\left[\left(\frac{1}{34}+\frac{1}{34}\right)\frac{3\times 3\times 5}{6.56}+\tan^{-1}\frac{3\times 3}{5\times 6.56}\right] = 0.01186P\ \text{[kip/ft}^2\text{]}$$

Stress σ_z at the middle of the clay layer:

Depth from bottom of footing to middle of clay layer = 8-3+8/2= 9 [ft]

$$R_1^2 = R_2^2 = 3^2+9^2 = 90$$
$$R_3 = \sqrt{3^2+3^2+9^2} = 9.95$$
$$\sigma_z = \frac{4\times P}{2\pi\times 6\times 6}\left[\left(\frac{1}{90}+\frac{1}{90}\right)\frac{3\times 3\times 9}{9.95}+\tan^{-1}\frac{3\times 3}{9\times 9.95}\right] = 497\times 10^{-5}P\ \text{[kip/ft}^2\text{]}$$

Stress σ_z at the bottom of the clay layer:

Depth from bottom of footing to bottom of clay layer = 8-3+8= 13 [ft]

$$R_1^2 = R_2^2 = 3^2+13^2 = 178$$
$$R_3 = \sqrt{3^2+3^2+13^2} = 13.67$$
$$\sigma_z = \frac{4\times P}{2\pi\times 6\times 6}\left[\left(\frac{1}{178}+\frac{1}{178}\right)\frac{3\times 3\times 13}{13.67}+\tan^{-1}\frac{3\times 3}{13\times 13.67}\right] = 261\times 10^{-5}P\ \text{[kip/ft}^2\text{]}$$

Average stress $\Delta\sigma_z$ in the clay layer by numerical integration using Simpson's rule:

$$\Delta\sigma_z = \frac{1}{2}\left[\frac{\sigma_z(Top)+\sigma_z(bottom)}{2}+\sigma_z(midell)\right]$$

$$\Delta\sigma_z = \frac{1}{2}\left[\frac{0.01186+261\times10^{-5}}{2}+497\times10^{-5}\right]P$$

$$\Delta\sigma_z = 610\times10^{-5}\,P \text{ [kip/ft}^2\text{]}$$

4.10.12.2.3 Over consolidated ratio of the clay layer OCR:

$$\sigma_o = 1.28 \text{ [kip/ft}^2\text{]}$$

$$\sigma_c = 1.6 \text{ [kip/ft}^2\text{]}$$

$$OCR = \frac{\sigma_c}{\sigma_o} = \frac{1.6}{1.28} = 1.25$$

4.10.12.2.4 Consolidation settlement of overly consolidated clay (σ_f<σ_c)

Column load P=40 [kib]
Considering a recompression index only, C_r=C_c=0.5 [-].
Average stress in the clay layer $\Delta\sigma_z = 610\times10^{-5}P = 611\times10^{-5}\times40 = 0.244$ [kip/ft^2]

$$\sigma_o = 1.28 \text{ [kip/ft}^2\text{]}$$

$$\sigma_c = 1.6 \text{ [kip/ft}^2\text{]}$$

$$\sigma_f = \sigma_o + \Delta\sigma_z = 1.28 + 0.244 = 1.52 \text{ [kip/ft}^2\text{]}$$

$$\sigma_f < \sigma_c$$

The consolidation settlement of overly consolidated clay layer (σ_f<σ_c) is obtained from:

$$Sc = \frac{C_r\,h}{1+e_o}\log\left(\frac{\sigma_f}{\sigma_o}\right)$$

where:

Sc Consolidation settlement, [ft].
h Layer thickness, [ft].
C_r Recompression index, [-].
e_o Initial void ratio, [-].
σ_o Overburden pressure in the middle of the layer, [kib/ ft^2].
σ_f Final stress on the clay layer, [kib/ ft^2].

The consolidation settlement is given by:

$$S_c = \frac{0.1\times 8}{1+0.7}\log\left(\frac{1.52}{1.28}\right) = 0.0351 \text{ [ft]}$$

$$S_c = 0.0351\times 12 = 0.42 \text{[in]}$$

4.10.12.2.5 Consolidation settlement of overly consolidated clay ($\sigma_f > \sigma_c > \sigma_o$)

Column load P= 80 [kib]
Considering compression and recompression indices, C_r=0.1 [-], C_c=0.5 [-].
Average stress in the clay layer $\Delta\sigma_z = 610\times 10^{-5} P = 611\times 10^{-5}\times 80 = 0.489 \text{ [kip/ft}^2\text{]}$

$$\sigma_o = 1.28 \text{ [kip/ft}^2\text{]}$$
$$\sigma_c = 1.6 \text{ [kip/ft}^2\text{]}$$
$$\sigma_f = \sigma_o + \Delta\sigma_z = 1.28 + 0.489 = 1.77 \text{ [kip/ft}^2\text{]}$$
$$\sigma_f > \sigma_c > \sigma_o$$

The consolidation settlement of overly consolidated clay layer ($\sigma_f > \sigma_c > \sigma_o$) is obtained from:

$$Sc = \frac{C_r h}{1+e_o}\log\left(\frac{\sigma_c}{\sigma_o}\right) + \frac{C_c h}{1+e_o}\log\left(\frac{\sigma_f}{\sigma_c}\right)$$

where:

Sc	Consolidation settlement, [ft].
h	Layer thickness, [ft].
C_r	Recompression index, [-].
C_c	Compression index, [-].
e_o	Initial void ratio, [-].
σ_o	Overburden pressure in the middle of the layer, [kib/ ft^2].
σ_c	Preconsolidation pressure of the layer, [kib/ ft^2].
σ_f	Final stress on the clay layer, [kib/ ft^2].

The consolidation settlement is given by:

$$S_c = \frac{0.1\times 8}{1+0.7}\log\left(\frac{1.6}{1.28}\right) + \frac{0.5\times 8}{1+0.7}\log\left(\frac{1.77}{1.6}\right) = 0.0456 + 0.1032 \text{ [ft]}$$

$$S_c = 0.0456\times 12 + 0.1032\times 12 = 0.55 + 1.24 = 1.79 \text{[in]}$$

4.10.12.3 Consolidation settlement by ELPLA

The consolidation settlements obtained from *GEO Tools* at the center of the footing are nearly the same as that of the hand calculation with a difference 0.04 [in]. Table 4.23 compares between settlements obtained from *GEO Tools* and those of the hand calculation at the center of the footing. The input data and results of *GEO Tools* are presented on the next pages.

Table 4.23 Settlement under the point load (stress is calculated a in the entire layer)

Clay behavior	Consolidation settlement [in]		Difference [in]
	Hand calculation S_c	*GEO Tools* S_c	
Overly consolidated clay layer (σ_f<σ_c)	0.42	0.42	0.00
Overly consolidated clay layer (σ_f >σ_c>σ_o)	1.79	1.83	0.04

```
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
************************************************************
Title: Consolidation settlement of overly consolidated clay (Sig_f < Sig_c)
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex11a

------------------------------------------------------------
Consolidation settlement in soil due to rectangular load
------------------------------------------------------------

Data:
Rectangular load                      q       [kip/ft2] = 1.11
Length                                a       [ft]      = 6.00
Width                                 b       [ft]      = 6.00

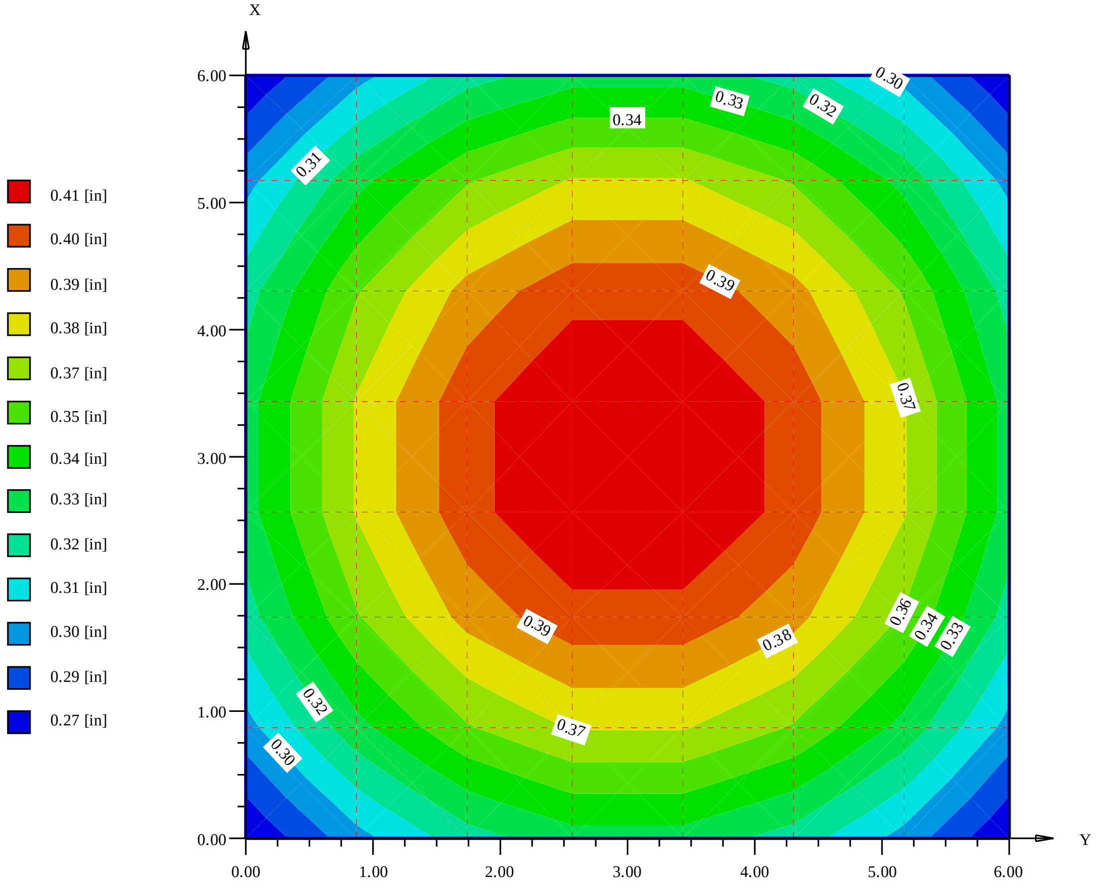
Soil Data:
Layer No.: 1
Compression index                     Cc      [-]       = 0.50
Recompression Index                   Cr      [-]       = 0.10
Over consolidated ratio               OCR     [-]       = 1.60
Initial void ratio                    eo      [-]       = 0.70
Unit weight                           Gamma_c [kip/ft3] = 0.08
Layer thickness                       h       [ft]      = 8.00


Overburden pressure                   Gamma*z [kip/ft2] = 0.96
Depth increment in z-direction        Dz      [ft]      = 0.00

Point coordinates:
X-coord.                              x       [ft]      = 3.00
Y-coord.                              y       [ft]      = 3.00
Y-coord.                              z       [ft]      = 5.00

Results:
Settlement
Layer No.: 1                          S1      [in]      = 0.42
Total                                 St      [in]      = 0.42
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [in]
Max. s = 0.41 at node 28, Min. s = 0.26 at node 57

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Scale: 13	**Project: User's Manual**
File: 4Con_Ex11a	**Date: 01-09-2017**
Page No.:	**Title: Consolidation settlement of overly consolidated clay (Sig_f < Sig_c)**

```
**********************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
**********************************************************
Title: Consolidation settlement of overly consolidated clay (Sig_f
>Sig_c>Sig_o)
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex11b

----------------------------------------------------------
Consolidation settlement in soil due to rectangular load
----------------------------------------------------------

Data:
Rectangular load                      q       [kip/ft2] = 2.22
Length                                a       [ft]      = 6.00
Width                                 b       [ft]      = 6.00

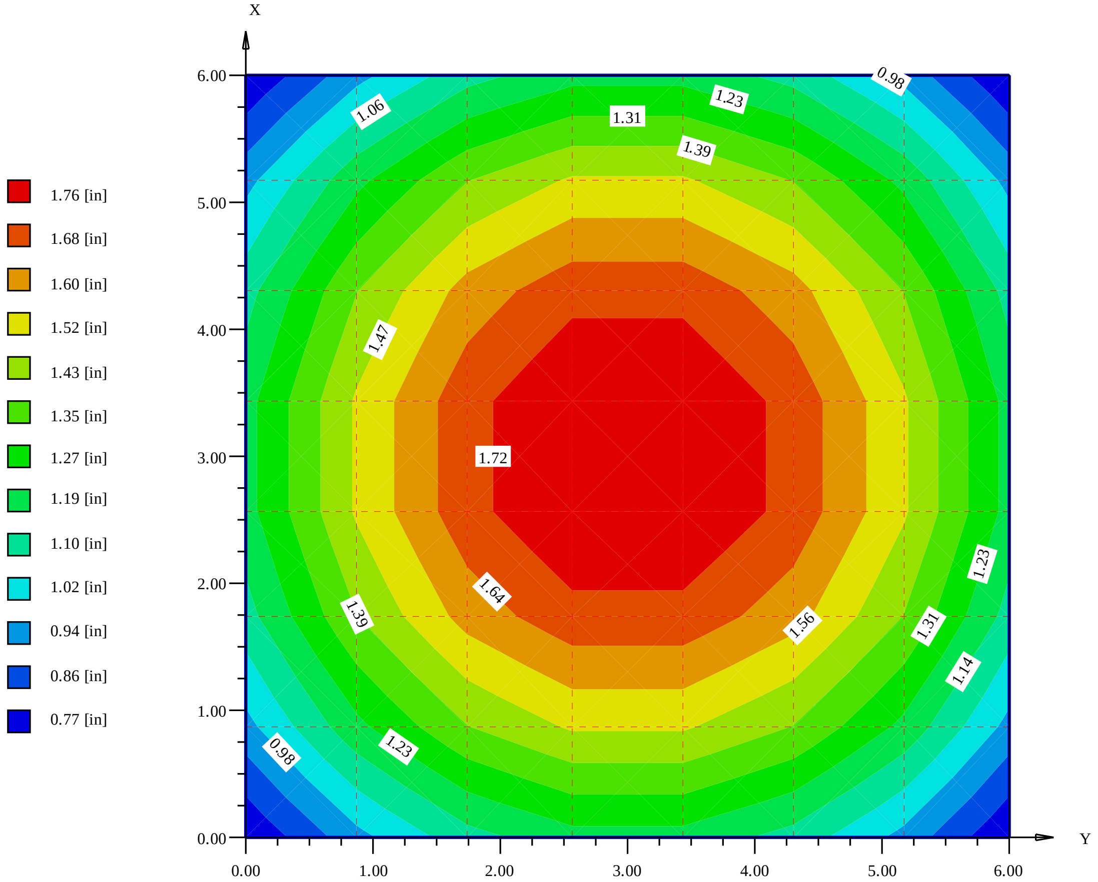
Soil Data:
Layer No.: 1
Compression index                     Cc      [-]       = 0.50
Recompression Index                   Cr      [-]       = 0.10
Over consolidated ratio               OCR     [-]       = 1.25
Initial void ratio                    eo      [-]       = 0.70
Unit weight                           Gamma_c [kip/ft3] = 0.08
Layer thickness                       h       [ft]      = 8.00


Overburden pressure                   Gamma*z [kip/ft2] = 0.96
Depth increment in z-direction        Dz      [ft]      = 0.00

Point coordinates:
X-coord.                              x       [ft]      = 3.00
Y-coord.                              y       [ft]      = 3.00
Y-coord.                              z       [ft]      = 5.00

Results:
Settlement
Layer No.: 1                          S1      [in]      = 1.83
Total                                 St      [in]      = 1.83
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [in]
Max. s = 1.81 at node 28, Min. s = 0.73 at node 57

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Page No.:	**Title: Consolidation settlement of overly consolidated clay (Sig_f >Sig_c>Sig_o)**

4.10.13Example 12: Consolidation settlement under an irregular loaded area

4.10.13.1 Description of the problem

To verify the consolidation settlement of an irregular loaded area calculated by the program *GEO Tools*, a hand calculation of a settlement under an L-shaped loaded area is compared with that obtained by *GEO Tools*.

An L-shaped loaded area of a load q = 150 [kN/m^2] is acting on a clay layer as shown in Figure 4.36. Find the settlement of the clay layer under the corner o of the L-shaped loaded area.

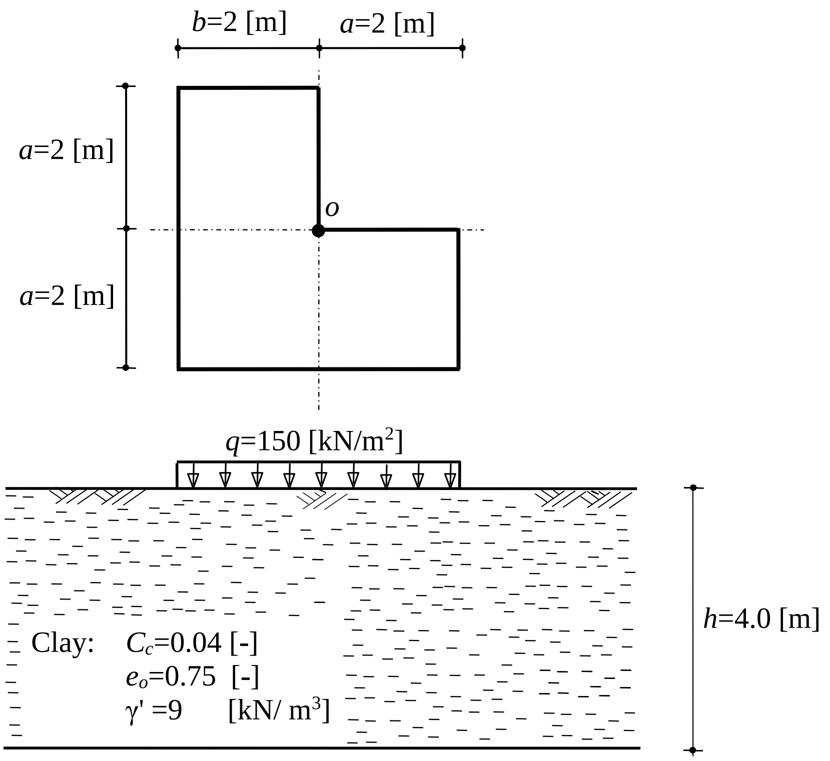


Figure 4.36 Soil profile under an L-shaped loaded area

4.10.13.2 Hand calculation

For a square loaded area of intensity q [kN/m^2] with side a [m] acting on the surface, the increment of vertical stress $\Delta\sigma_{av}$ [kN/m^2] in the soil layer of thickness h under the corner of the square can be expressed as:

$$\Delta\sigma_{av} = \frac{q}{2\pi\, h}\left\{\left(2a \ln \frac{(c-a)\,(m+a)}{(c+a)\,(m-a)} + h\tan^{-1}\frac{a^2}{h.c}\right)\right\}$$

where:

$m = \sqrt{2a^2}$ and $c = \sqrt{2\,a^2 + h^2}$

h Thickness of the soil layer, [m].

The average stress $\Delta\sigma_{va}$ in the entire clay layer under the corner *o* of L-shaped loaded area is:

$$m = \sqrt{2a^2} = \sqrt{2 \times 2^2} = 2.83$$
$$c = \sqrt{2\,a^2 + h^2} = \sqrt{2 \times 2^2 + h^2} = 4.9$$

$$\Delta\sigma_{av} = \frac{3 \times 150}{2\pi\, 4}\left\{\left(2 \times 2\ln\frac{(4.9-2)\,(2.83+2)}{(4.9\ +2)\,(2.83\ -2)}\ \ + 4\tan^{-1}\frac{2^2}{4 \times 4.9}\right)\right\}$$

$$\Delta\sigma_{av} = 78.44$$

Overburden stress σ_o at the middle of the clay layer (z=2 [m]):

$$\sigma_o = \gamma' z_2 = 9 \times 2 = 18\ \ [\text{kN/m}^2]$$

Settlement S_c of the clay layer:

$$S_c = \frac{C_c H}{1+e_o}\log\frac{\Delta\sigma_{va} + \sigma_o}{\sigma_o} = \frac{0.04 \times 4}{1+0.75}\log\frac{78.44+18}{18} = 0.0667\ \ [\text{m}] = 6.67\,[\text{cm}]$$

4.10.13.3 Settlement by GEO Tools

The settlement obtained from *GEO Tools* at the center *c* of the circular loaded area is 6.47 [cm]. It is nearly same as that of the hand calculation. The input data and results of *GEO Tools* are presented on the next pages.

```
**********************************************************
                        GEO Tools
                         Version 10
     Program authors Prof. M. El Gendy/ Dr. A. El Gendy
**********************************************************
Title: Settlement of a circular loaded area resting on a thin clay layer
Date: 01-09-2017
Project: User's Manual
File: 4Con_Ex12

----------------------------------------------------------
Consolidation settlement in soil due to polygon load
----------------------------------------------------------

Data:
Rectangular load                      q        [kN/m2] = 150.00

Soil Data:
Layer No.: 1
Compression index                     Cc       [-]     = 0.04
Recompression Index                   Cr       [-]     = 0.04
Over consolidated ratio               OCR      [-]     = 1.00
Initial void ratio                    eo       [-]     = 0.75
Unit weight                           Gamma_c  [kN/m3] = 9.00
Layer thickness                       h        [m]     = 4.00

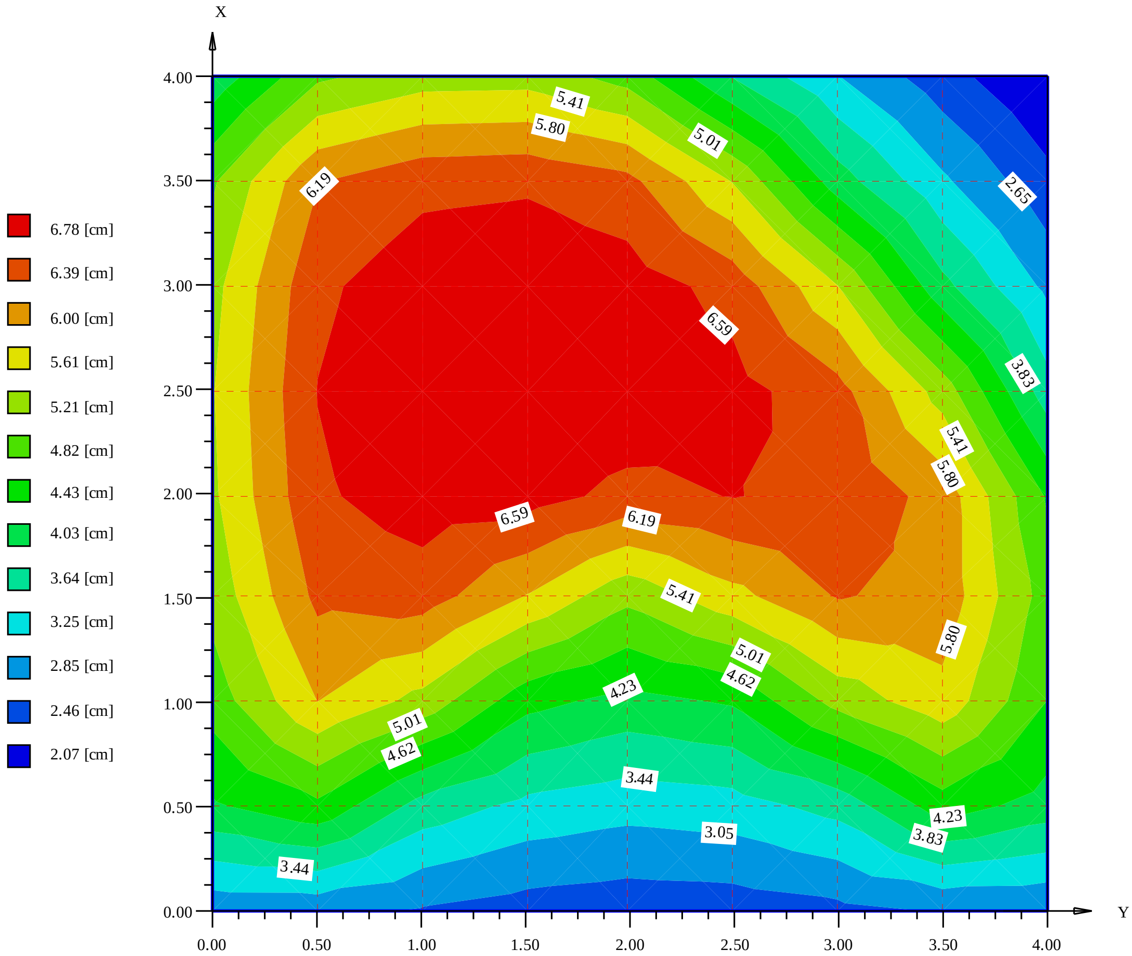

Overburden pressure                   Gamma*z  [kN/m2] = 0.00
Depth increment in z-direction        Dz       [m]     = 0.00

Point coordinates:
X-coord.                              x        [m]     = 2.00
Y-coord.                              y        [m]     = 2.00
Y-coord.                              z        [m]     = 0.00

Results:
Settlement
Layer No.: 1                          S1       [cm]    = 6.47
Total                                 St       [cm]    = 6.47

Coord.(Polygon load):
---------------------------
No.      Coord.      Coord.
  I           x           y
[-]         [m]         [m]
---------------------------
  1        4.00        2.00
  2        2.00        4.00
  3        0.00        4.00
  4        0.00        0.00
  5        2.00        2.00
  6        4.00        0.00
---------------------------
```

Method (9) (Layered soil model)
Flexible Foundation



Settlements s [cm]
Max. s = 6.98 at node 49, Min. s = 1.87 at node 81

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Scale: 27	**Project: User's Manual**
File: 4Con_Ex12	**Date: 01-09-2017**
Page No.:	**Title: Settlement of a circular loaded area resting on a thin clay layer**

4.11 References

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